Discussion Section 7

- Viterbi algorithm review
  - Homework 6 questions

- GENSCAN model

- Amortized analysis (for asymptotics)

Viterbi training

Viterbi training

- First step:
  - Find the most probable sequence of states

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Sequence	A	T	T	C	A	G	C	A

Viterbi training

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AT-rich								

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Sequence	A	T	T	C	A	G	C	A
AT-rich	●	●	●	●	●	●	●	●

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Sequence	A	T	T	C	A	G	C	A
AT-rich	●	●	●	●	●	●	●	●
GC-rich								

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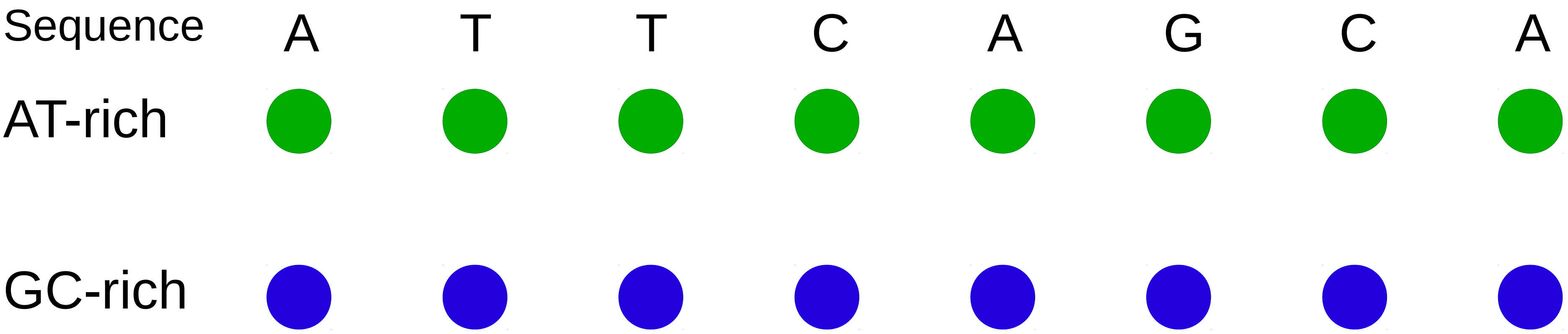


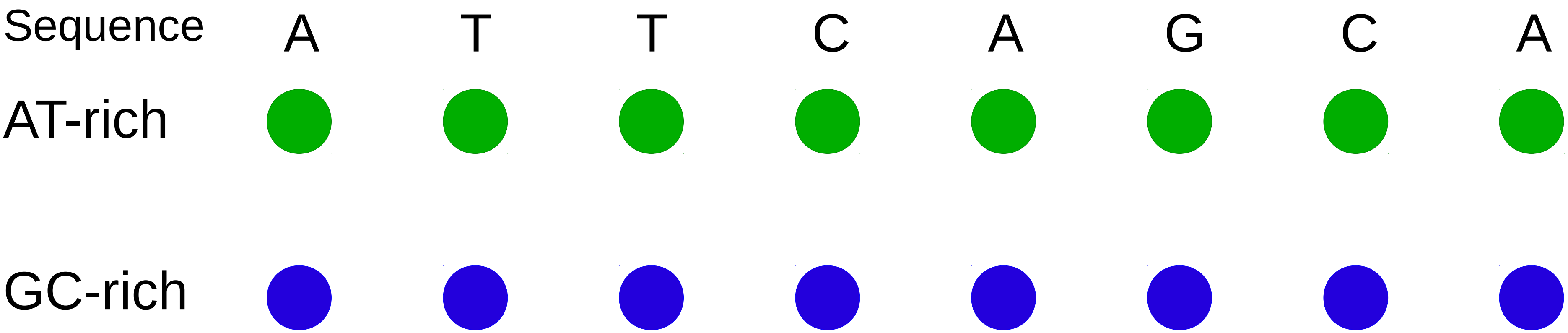
Sequence A T T C A G C A

AT-rich

GC-rich

Viterbi training

- First step:
  - Find the most probable sequence of states
    - This depends on the specified probabilities



Sequence A T T C A G C A

AT-rich

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  - Find the most probable sequence of states
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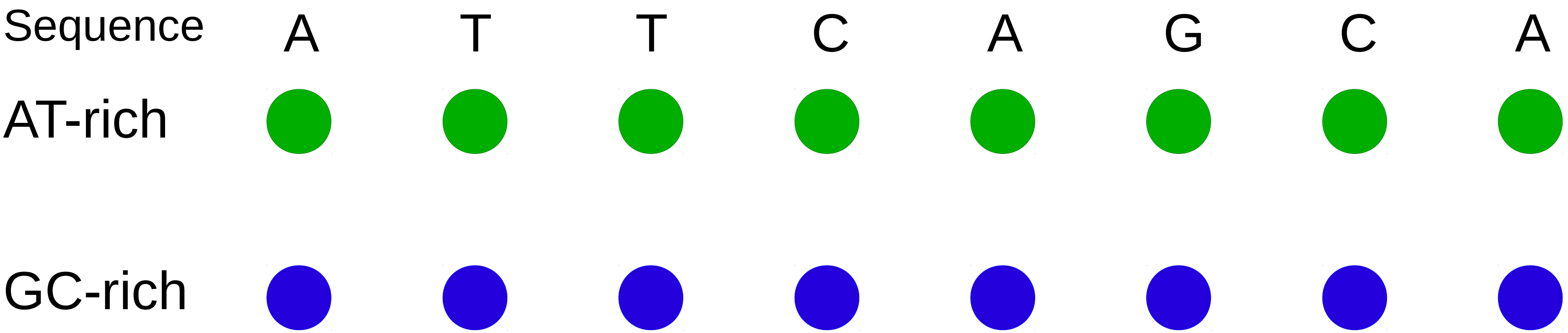

Viterbi training

- First step:
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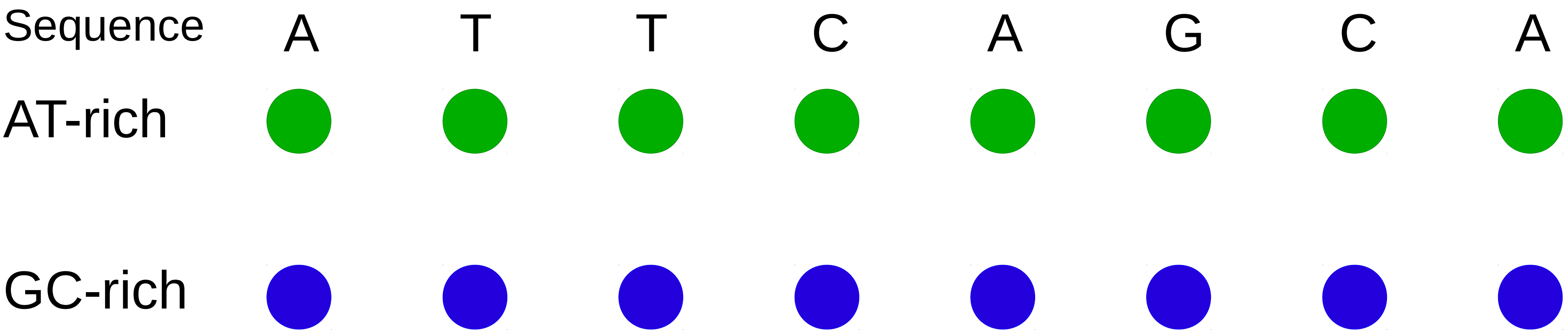

Sequence A T T C A G C A

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      - Transition
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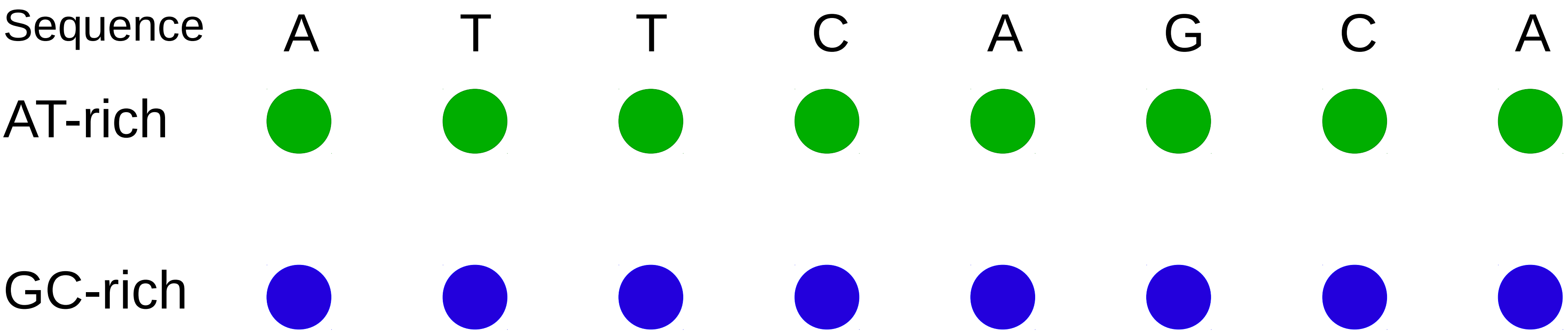

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- First step:
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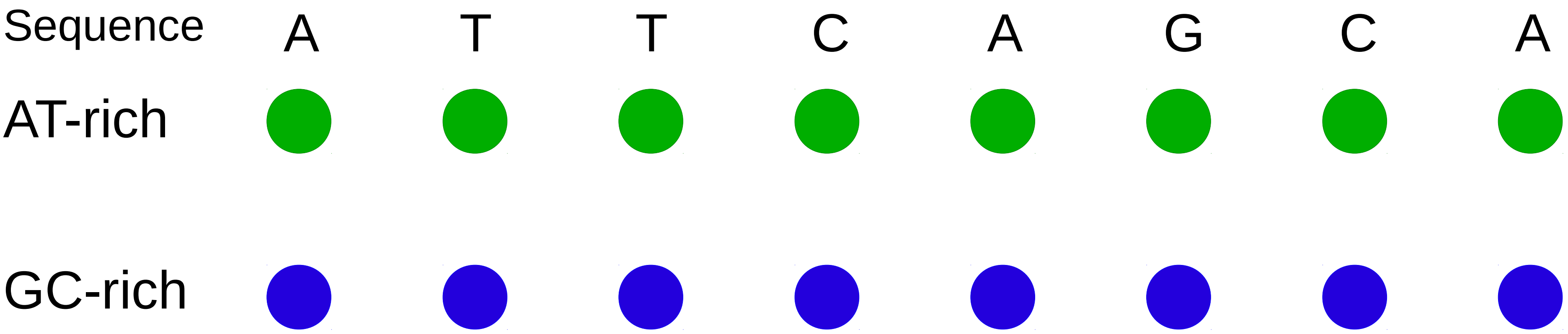
    - What is prioritized?



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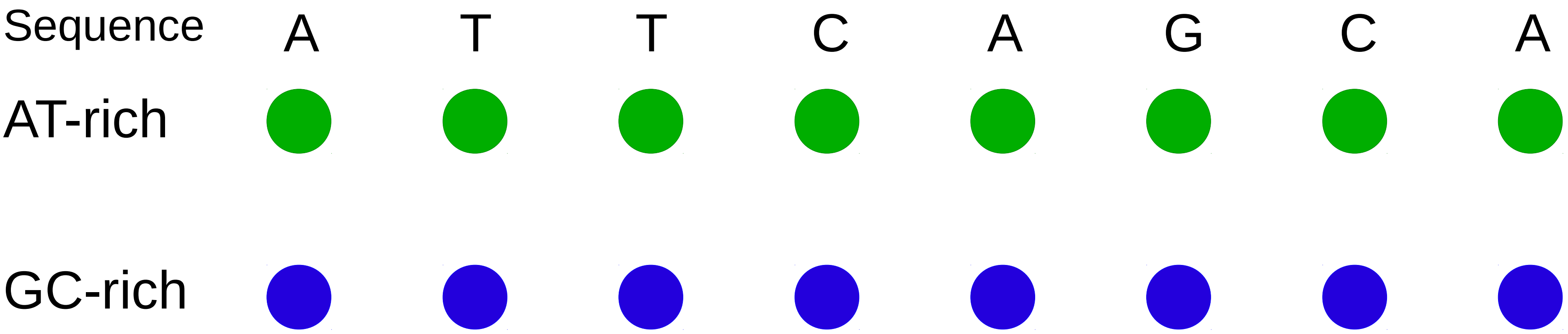

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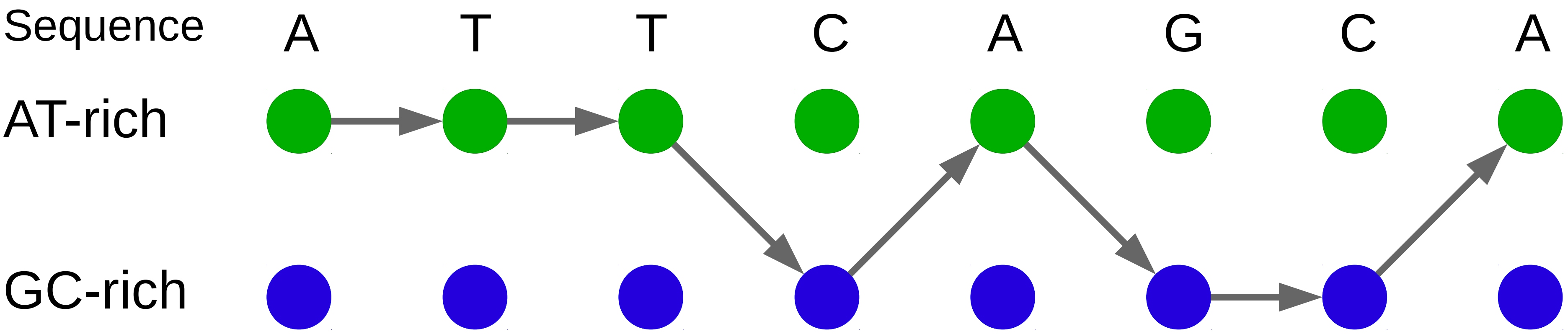

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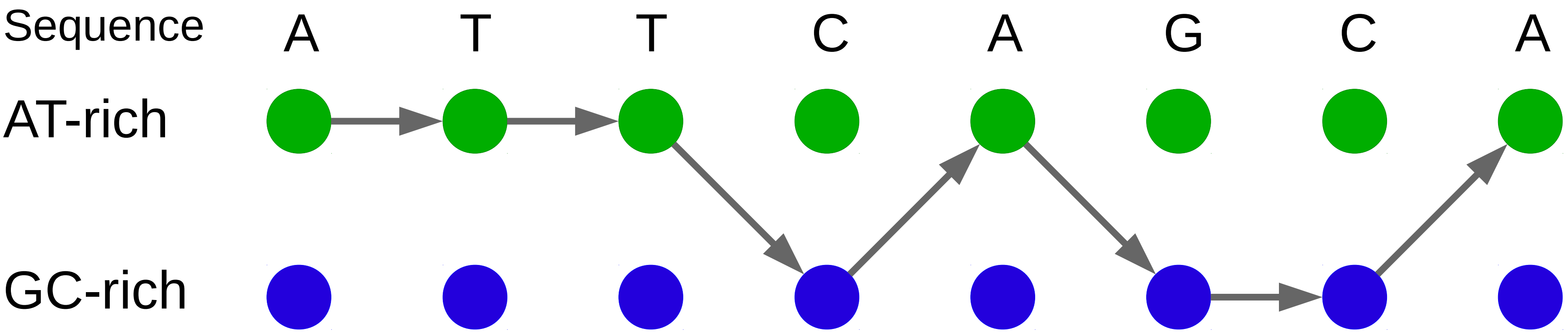

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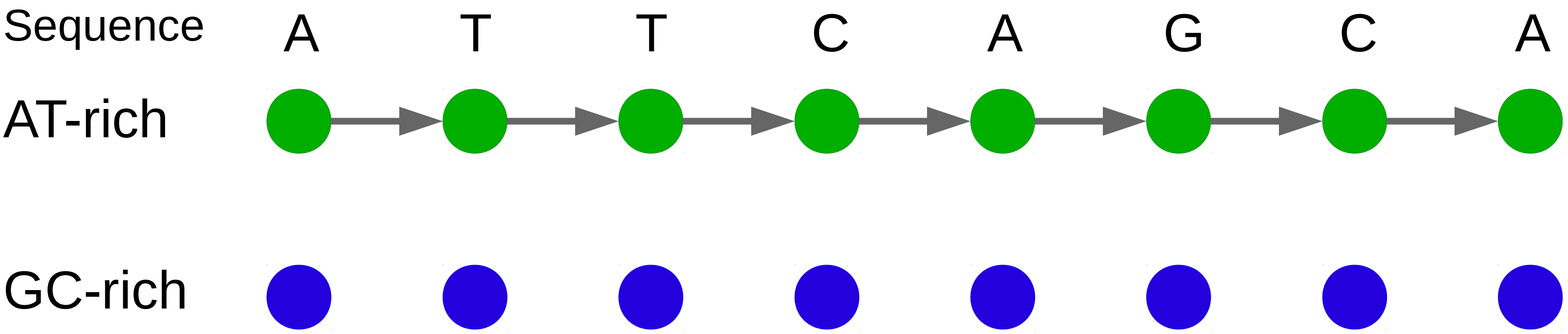

Sequence A T T C A G C A

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      - Correct emissions
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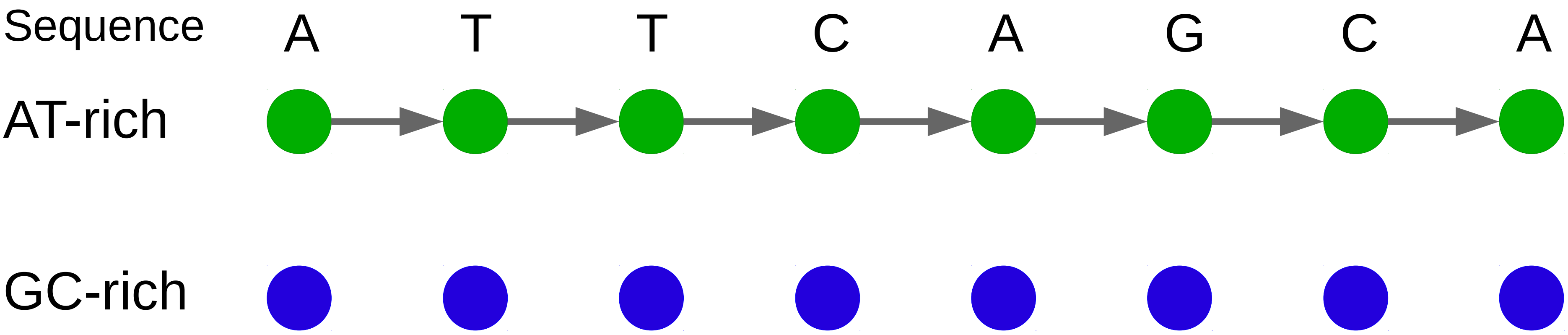
      - Balance between the two



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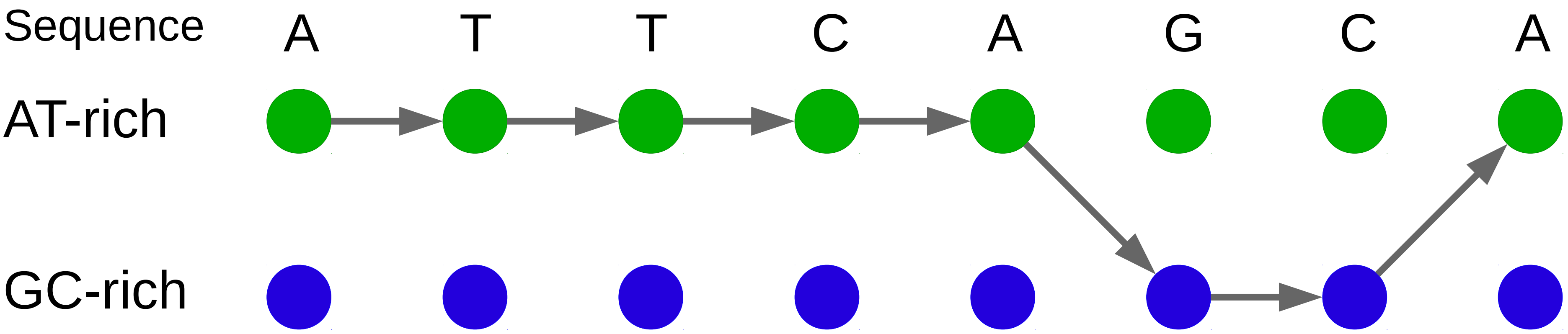
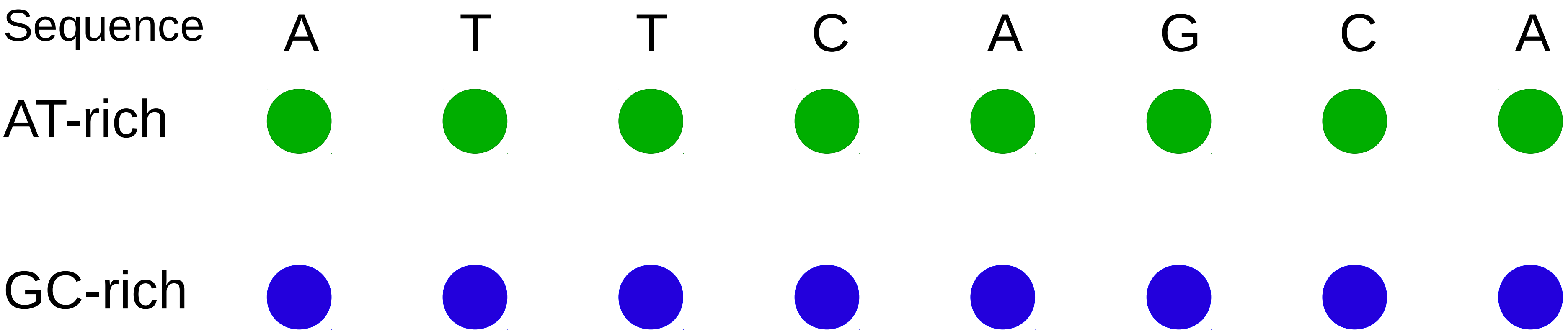


Viterbi training

- First step:
  - Find the most probably sequence of states
    - How do we actually determine the path?



Sequence A T T C A G C A

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Viterbi training

- First step:
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    - How do we actually determine the path?
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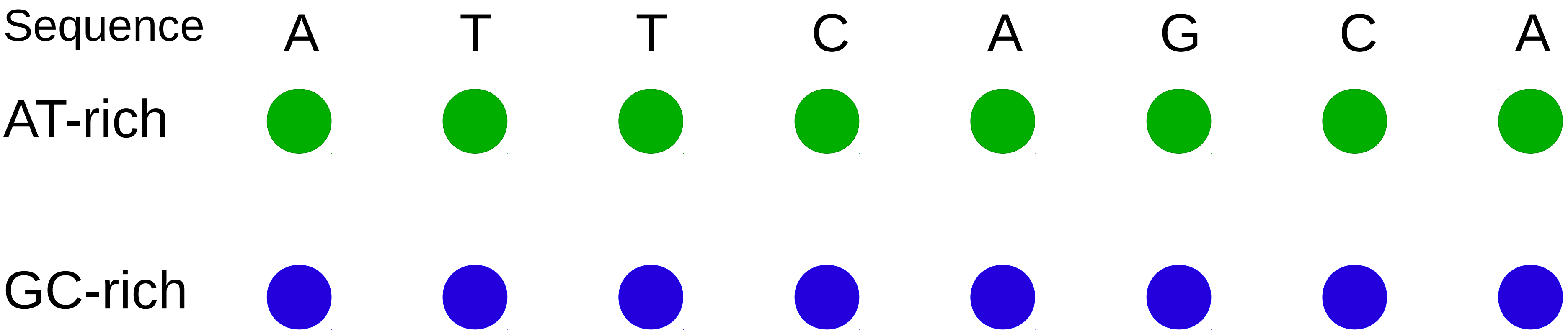

Viterbi training

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  - Find the most probably sequence of states
    - How do we actually determine the path?
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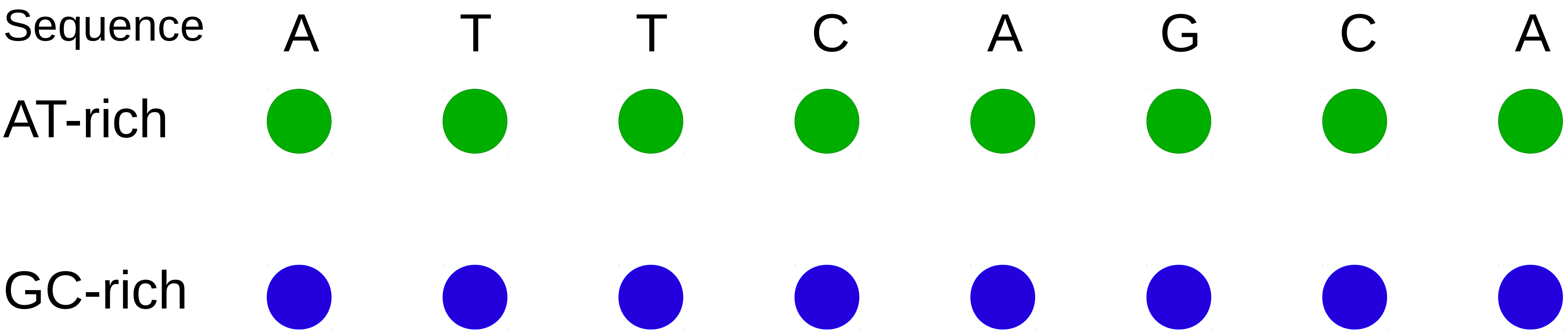
      - What's the most probable path that uses state *j* at sequence position *i*?



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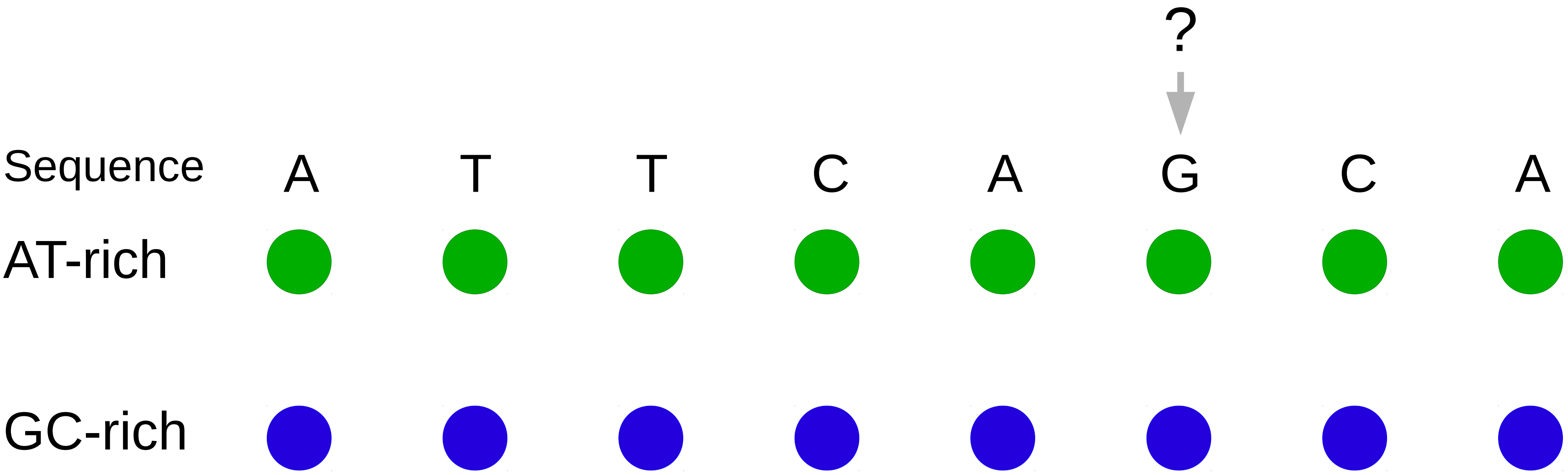

?

Sequence	A	T	T	C	A	G	C	A
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GC-rich	●	●	●	●	●	●	●	●

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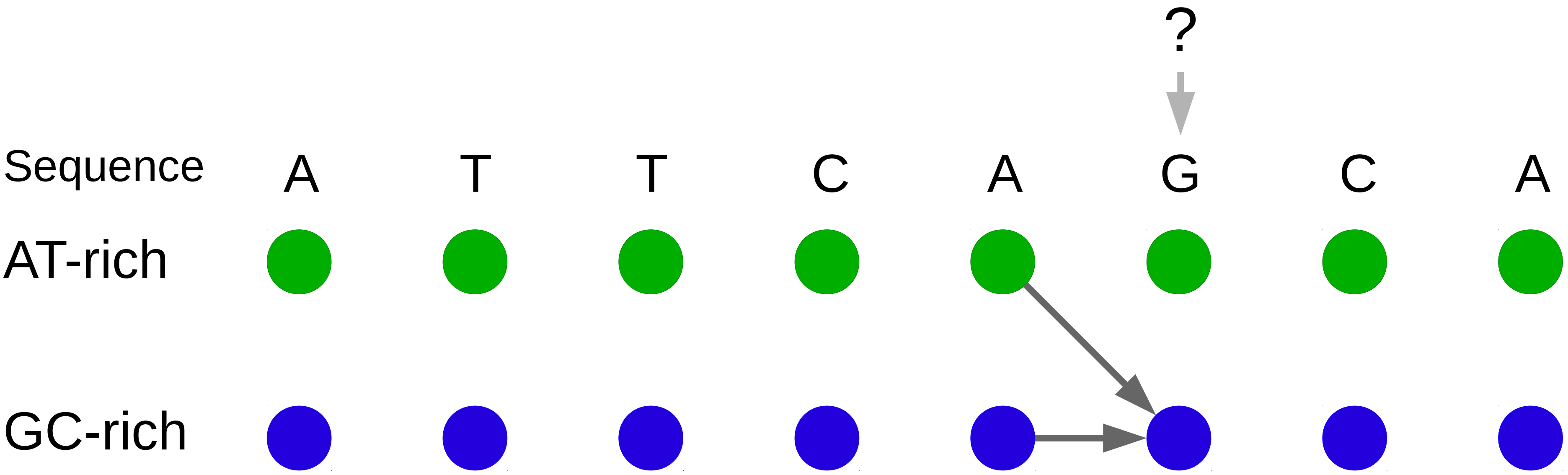

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      - What's the most probable path that uses state *j* at sequence position *i*?
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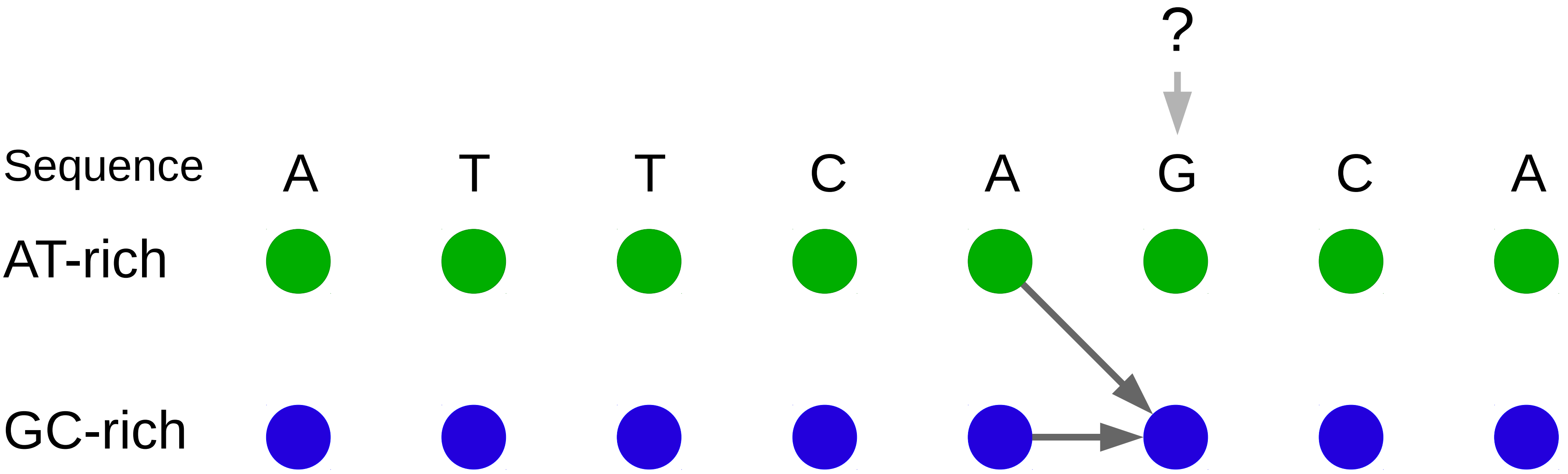

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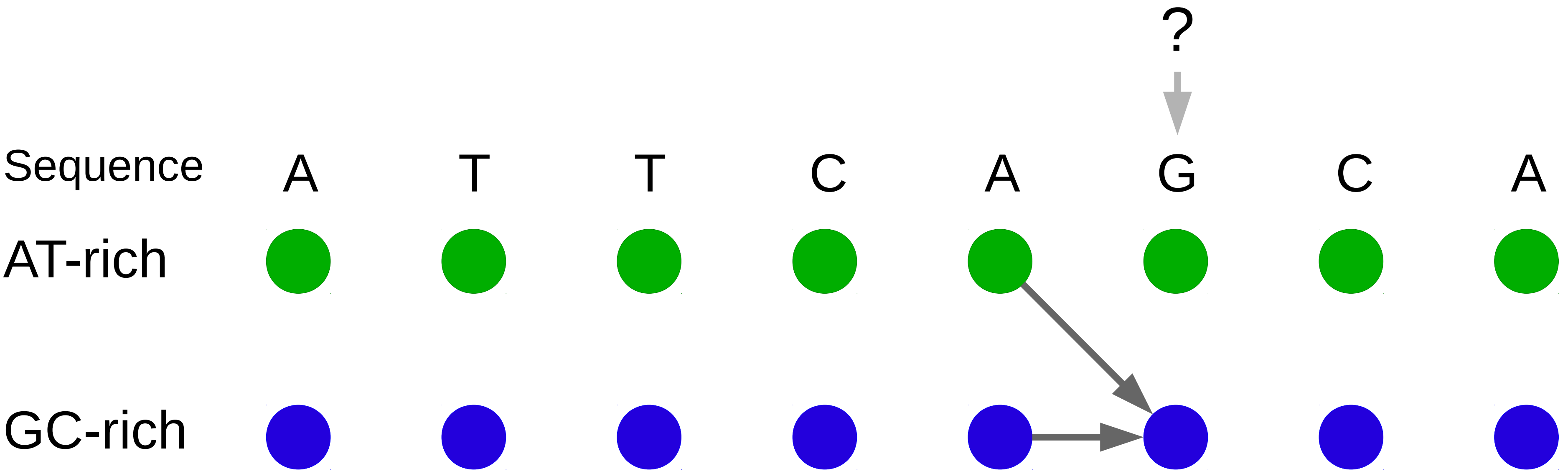
      - Trace back the most probable path at the end



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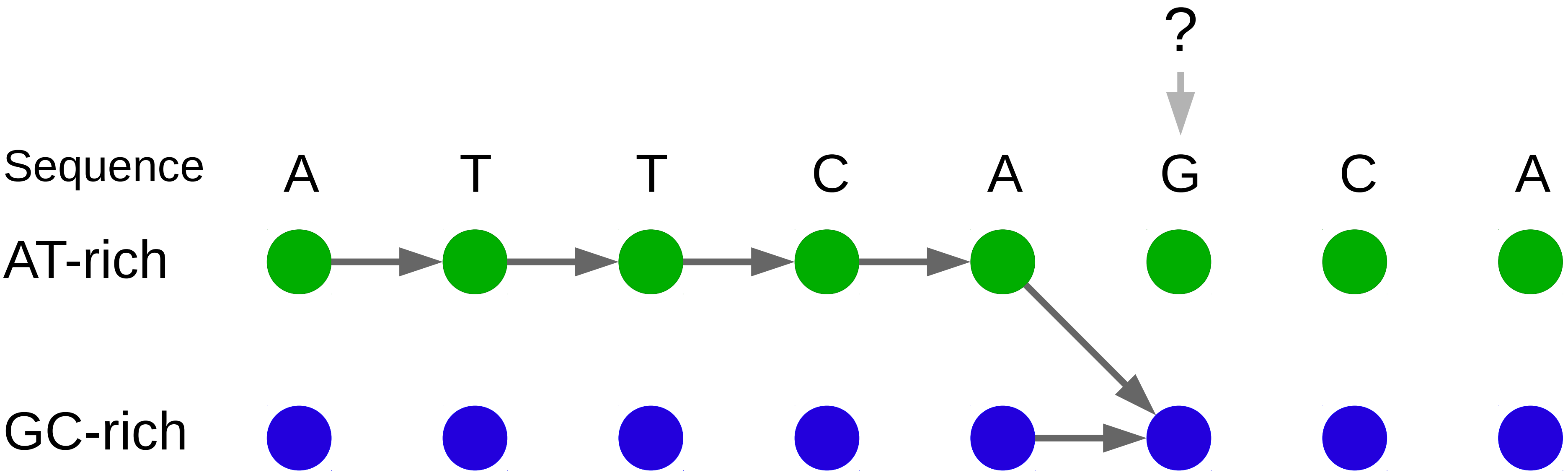

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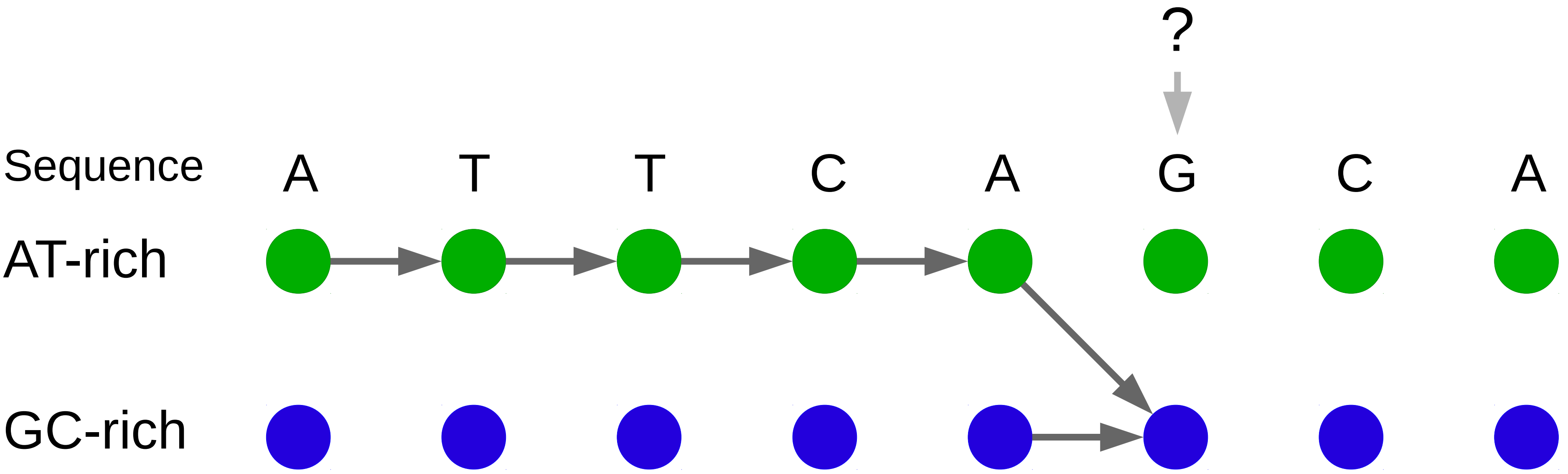
      - Trace back the most probable path at the end
      - Aren't you glad you've already written this algorithm?



Viterbi training

- Second step:
  - Update probabilities according to current most probable path

Viterbi training

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      - Set the transition probabilities to those frequencies for the next step

Viterbi training

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  - Update probabilities according to current most probable path
    - For the homework, you'll only be updating the transition probabilities
      - Calculate how often transitions happen between all pairs of states
      - Set the transition probabilities to those frequencies for the next step
      - Do this 10 times

Homework 5 tips

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    - What about log(0)?

Homework 5 tips

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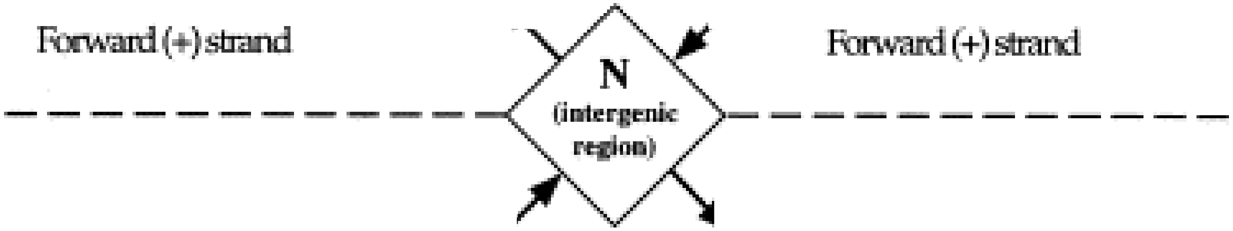
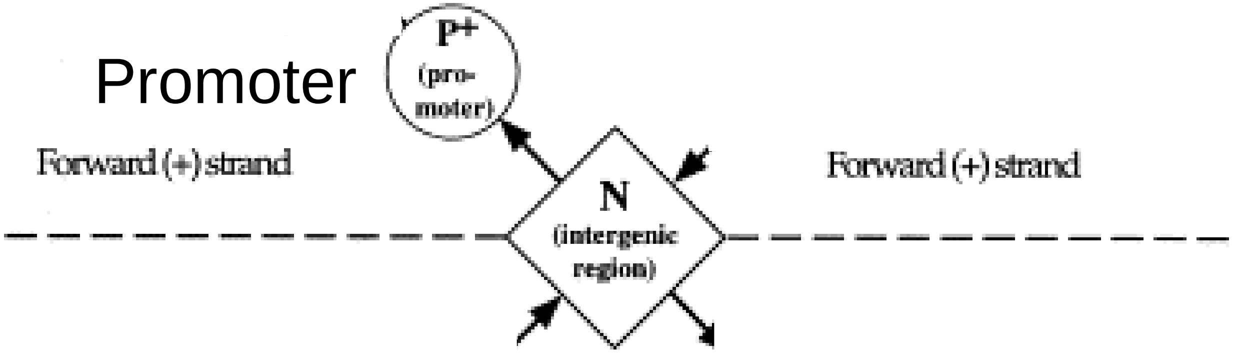
  - Take the logarithm
  - Add log-transformed probabilities instead of multiplying
    - What about log(0)?
  - See Mann and Rabiner (pg 273) for more details (link on website)

GENSCAN model

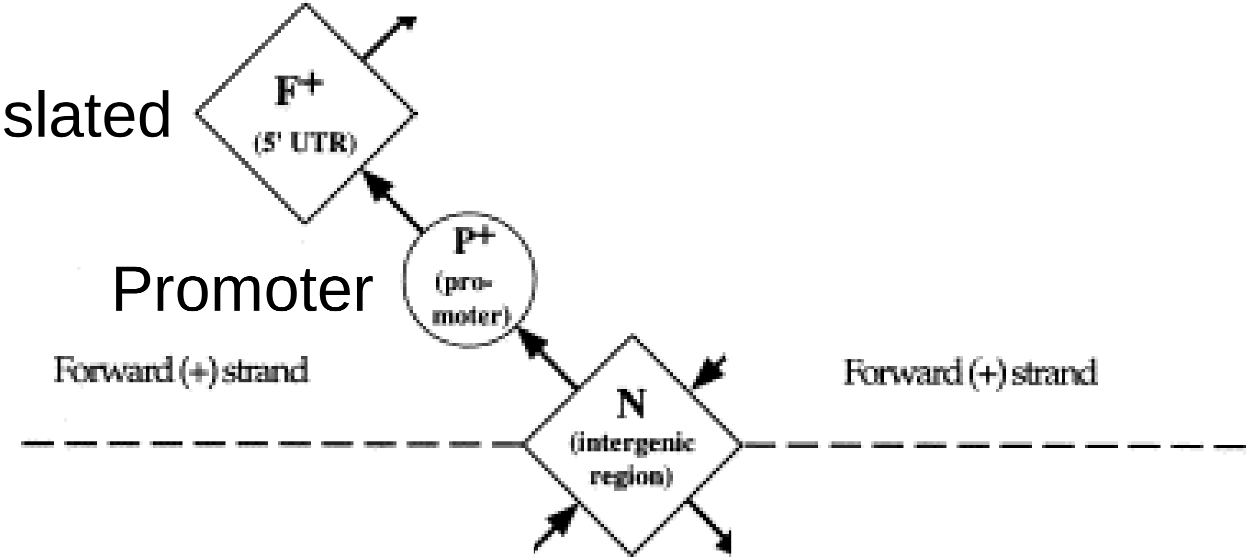


GENSCAN model



Promoter

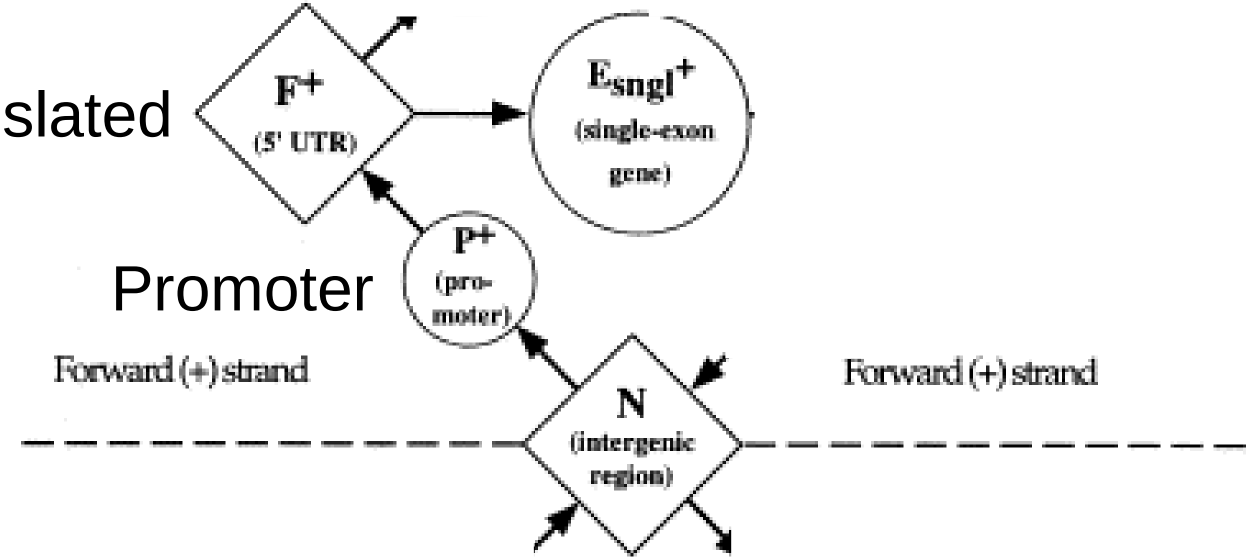
GENSCAN model



5' untranslated

Promoter

GENSCAN model

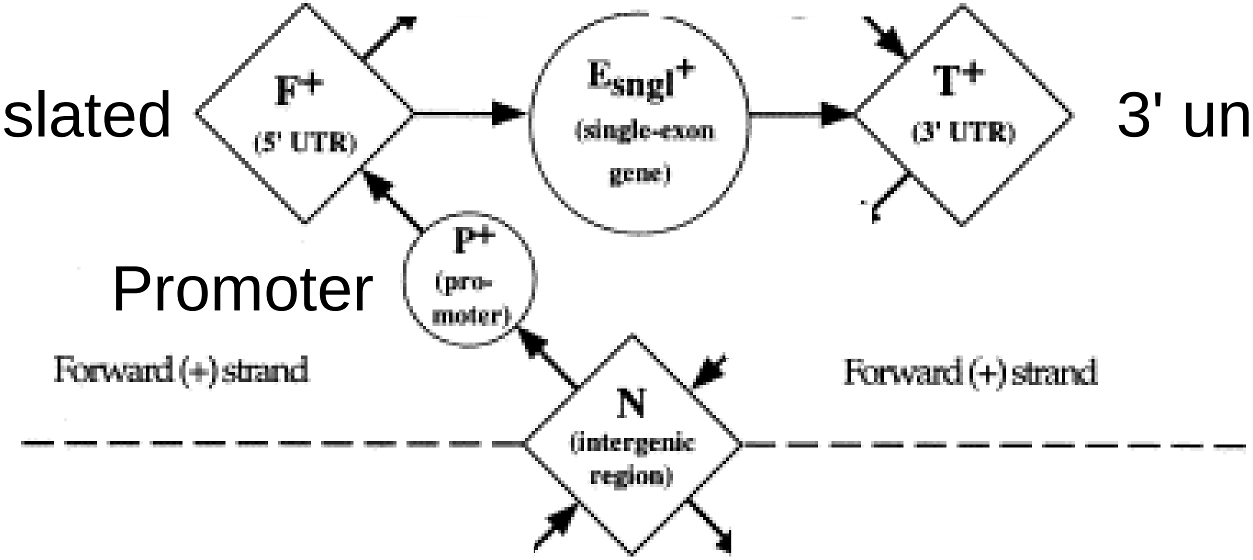


5' untranslated

Promoter

GENSCAN model

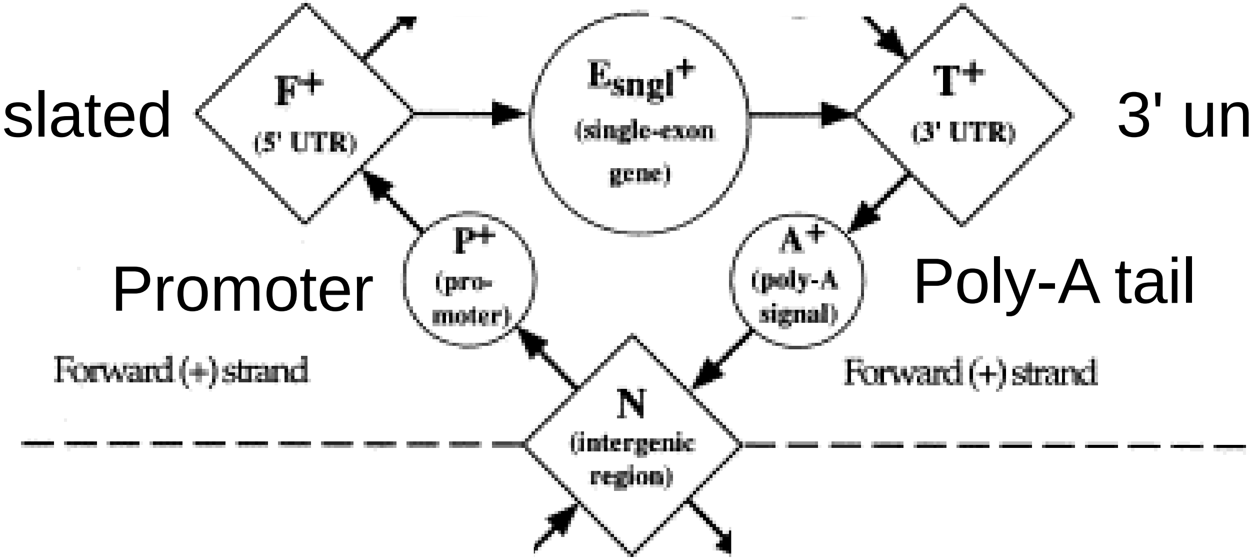
5' untranslated



3' untranslated

Promoter

GENSCAN model



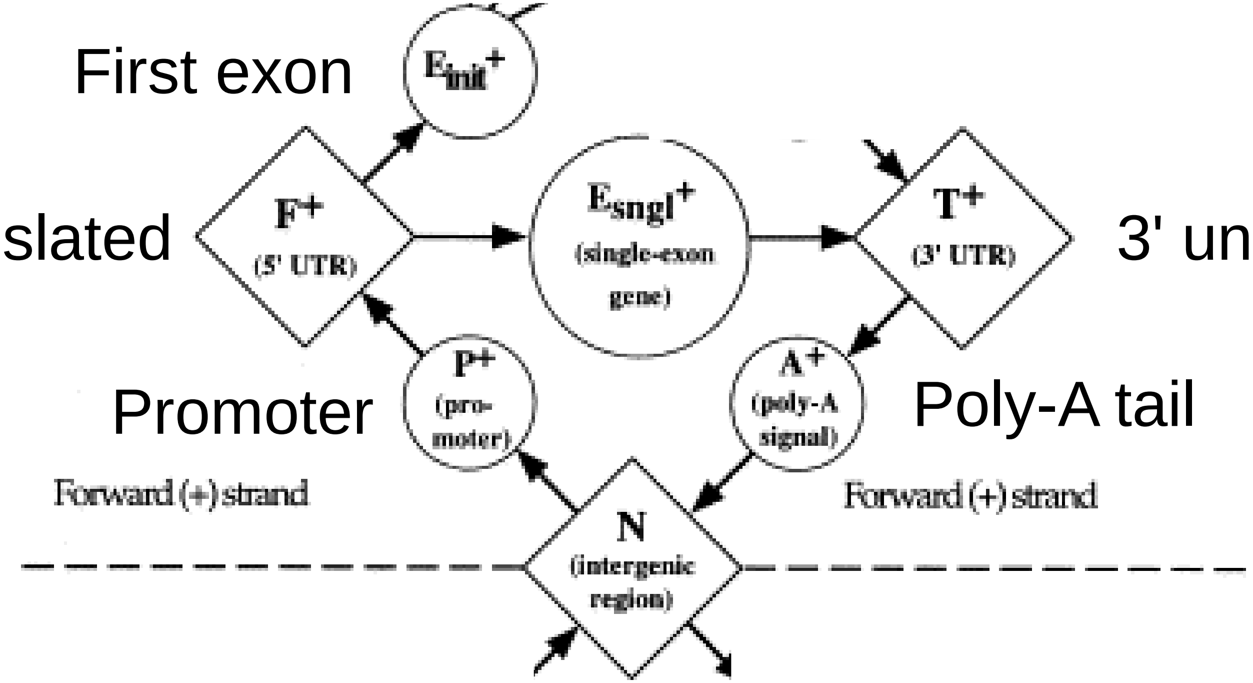
5' untranslated

3' untranslated

Promoter

Poly-A tail

GENSCAN model



First exon

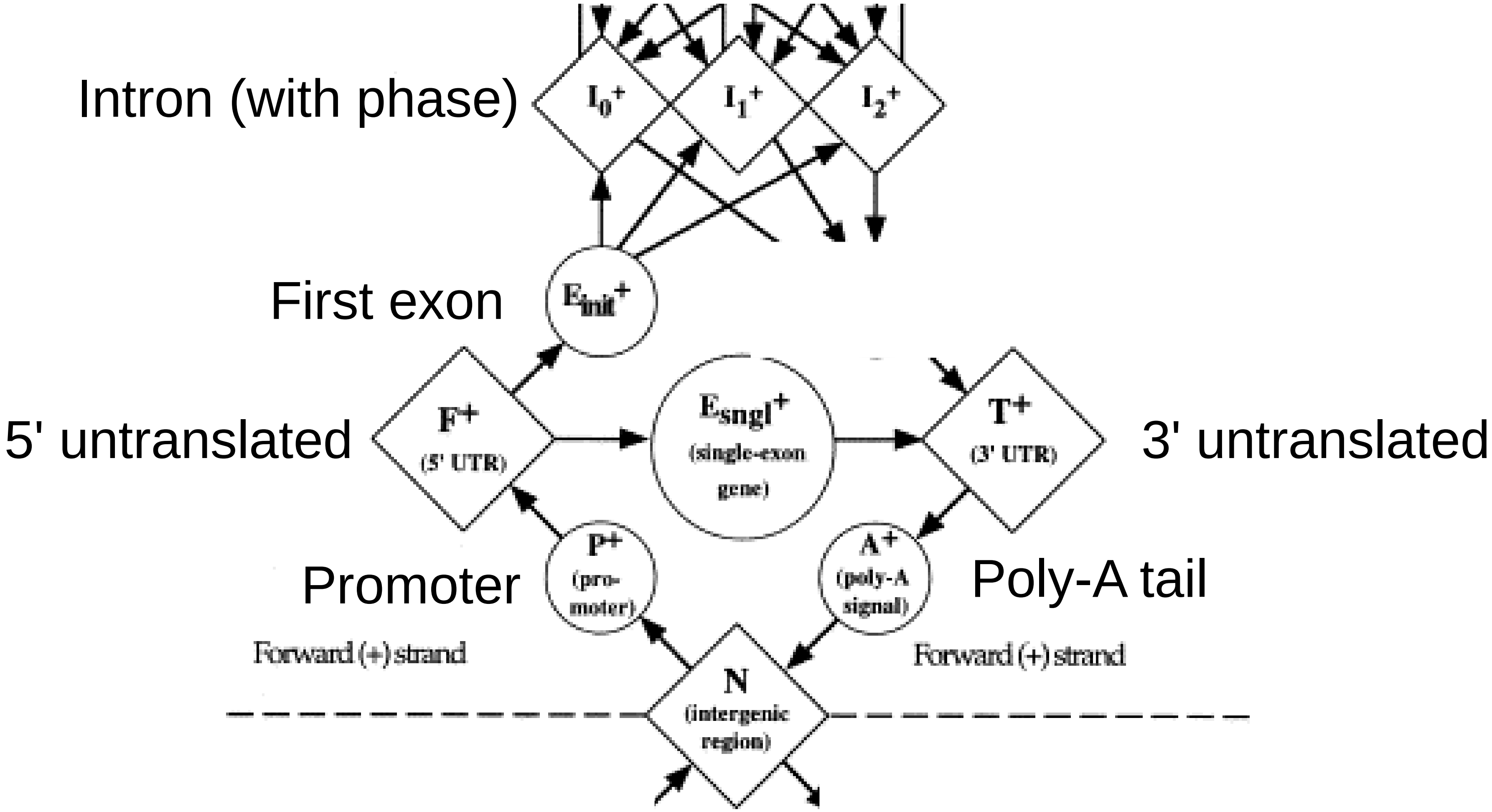
5' untranslated

3' untranslated

Promoter

Poly-A tail

GENSCAN model



Intron (with phase)

First exon

5' untranslated

3' untranslated

Promoter

Poly-A tail

GENSCAN model

Intron (with phase)

Intron (with phase)

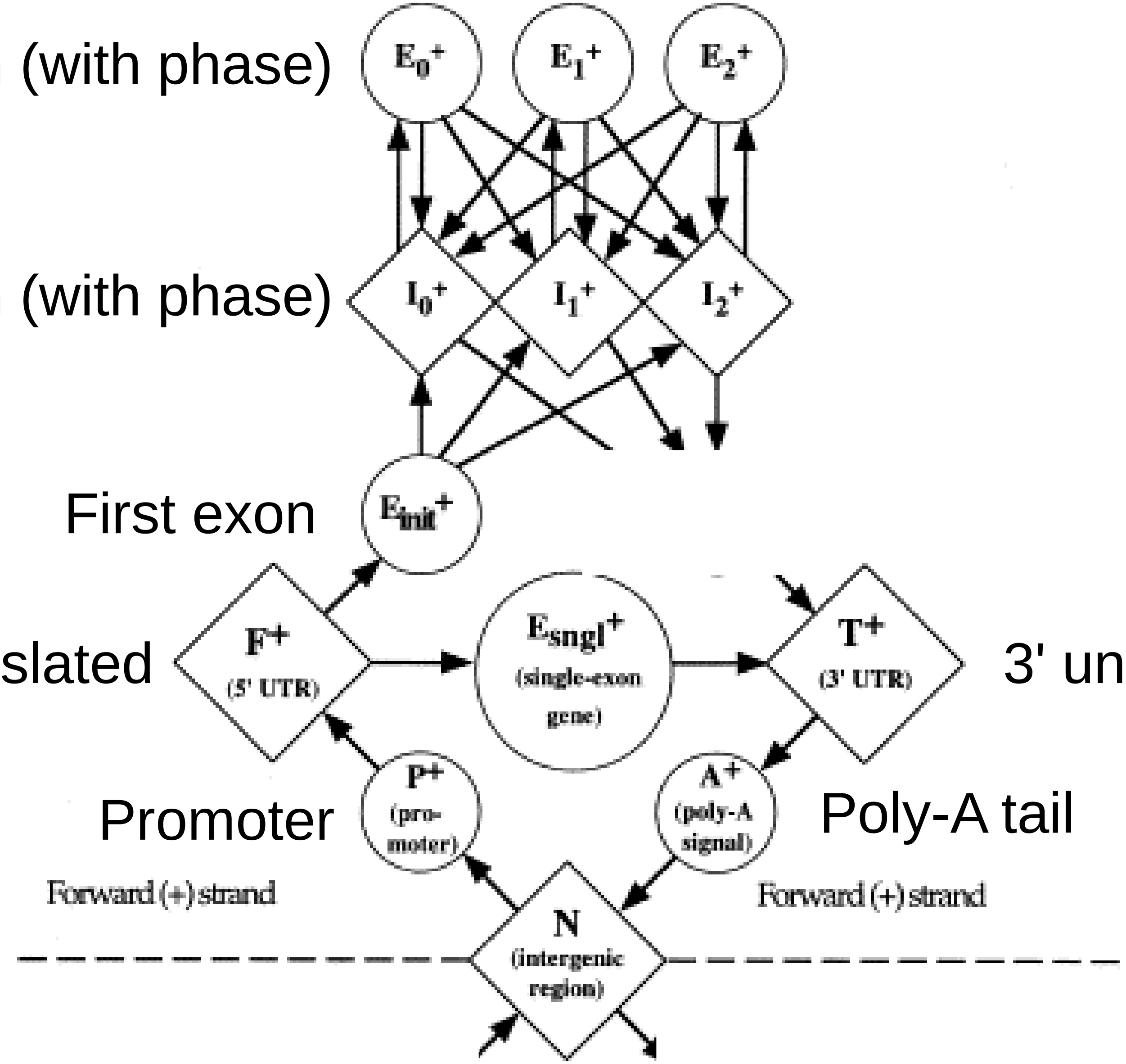
First exon

5' untranslated

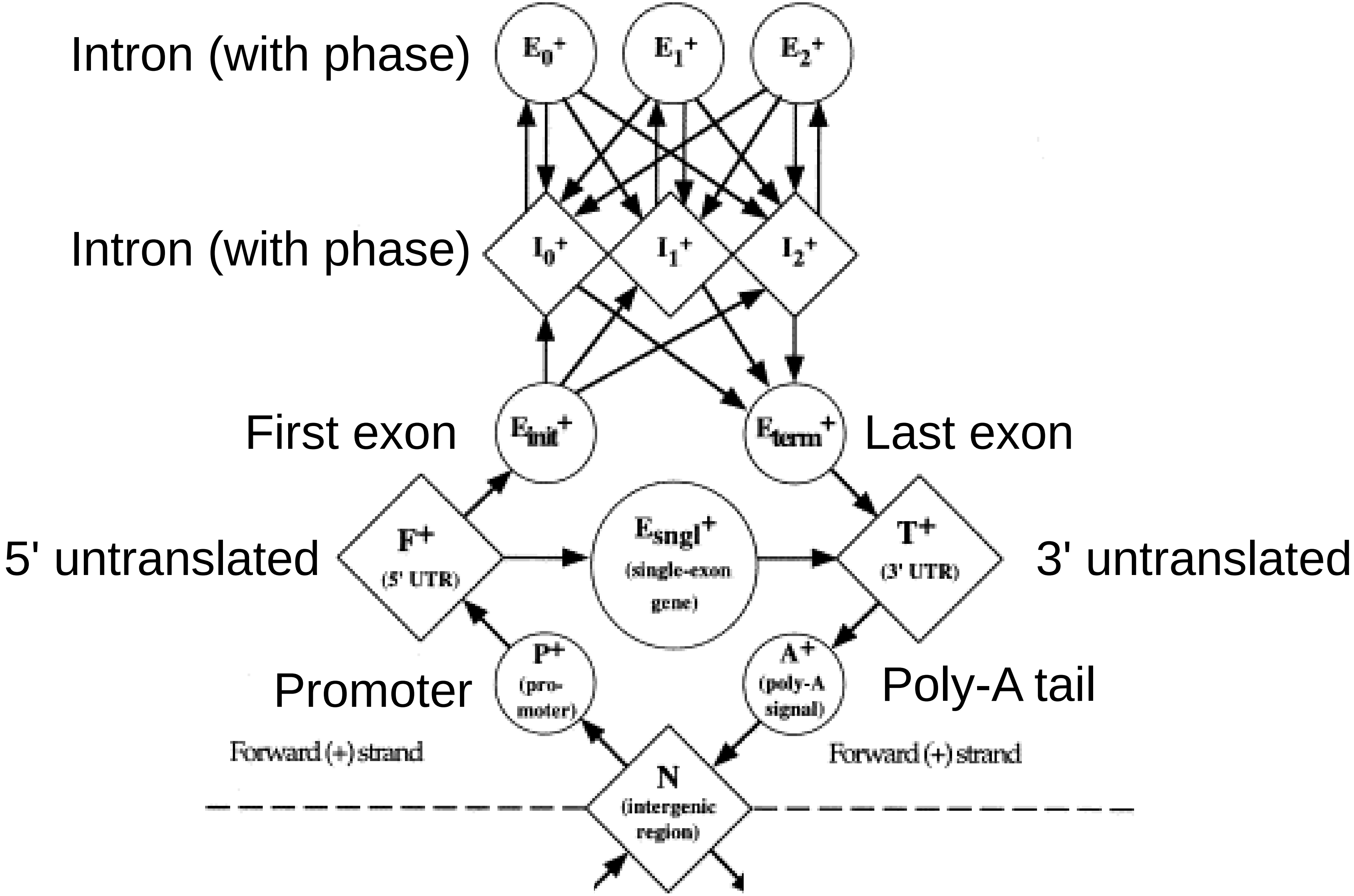
3' untranslated

Promoter

Poly-A tail



GENSCAN model



Intron (with phase)

Intron (with phase)

First exon

Last exon

5' untranslated

3' untranslated

Promoter

Poly-A tail

GENSCAN details

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- Exons modeled with 5th-order HMM
  - Corresponds to hexamer probabilities

Amortized analysis

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  - Example: arrays with dynamic resizing

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- 3 different classes (all closely related)
  - Aggregate analysis
  - Accounting method

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Amortized analysis example: aggregate analysis

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- Implementing a queue (first in, first out) with two stacks (first in, last out)

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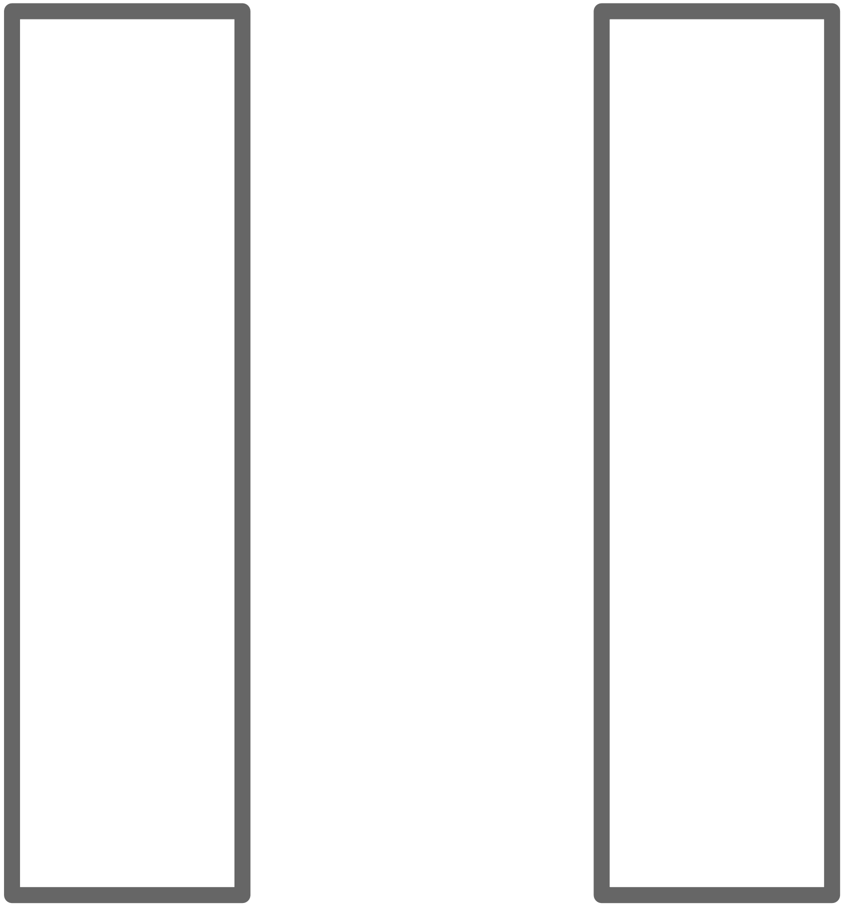

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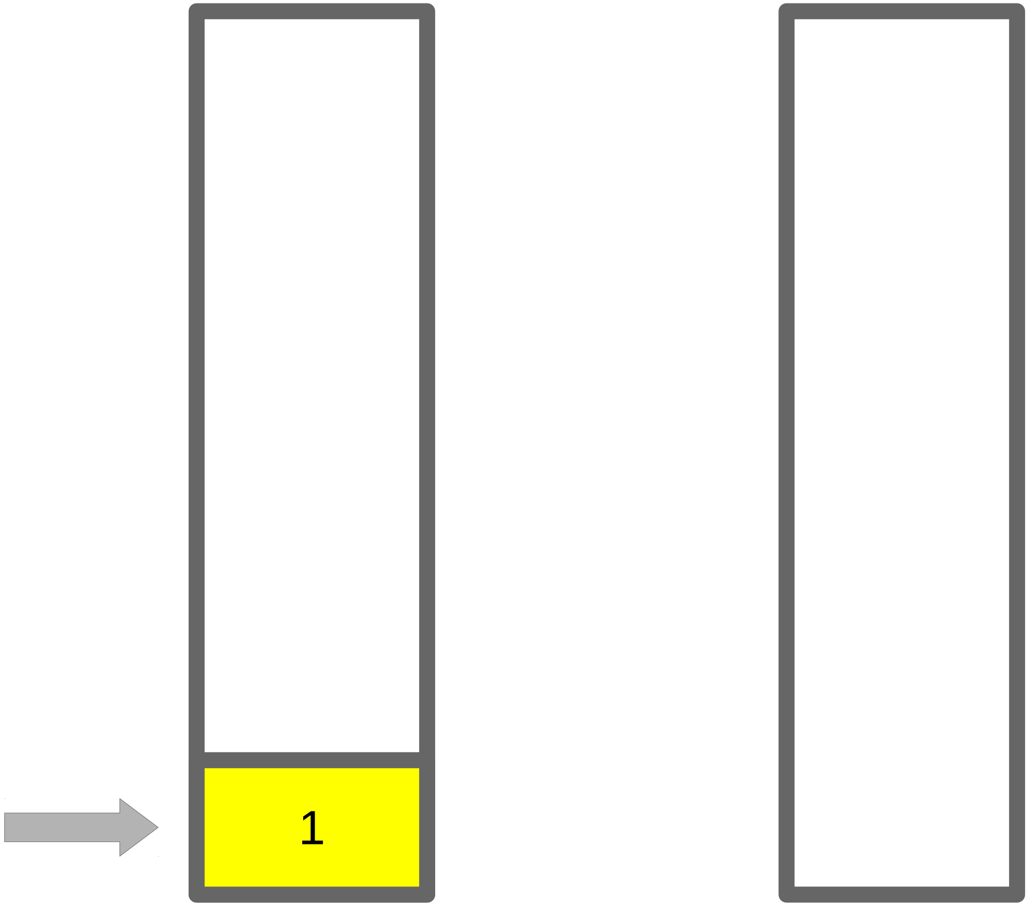

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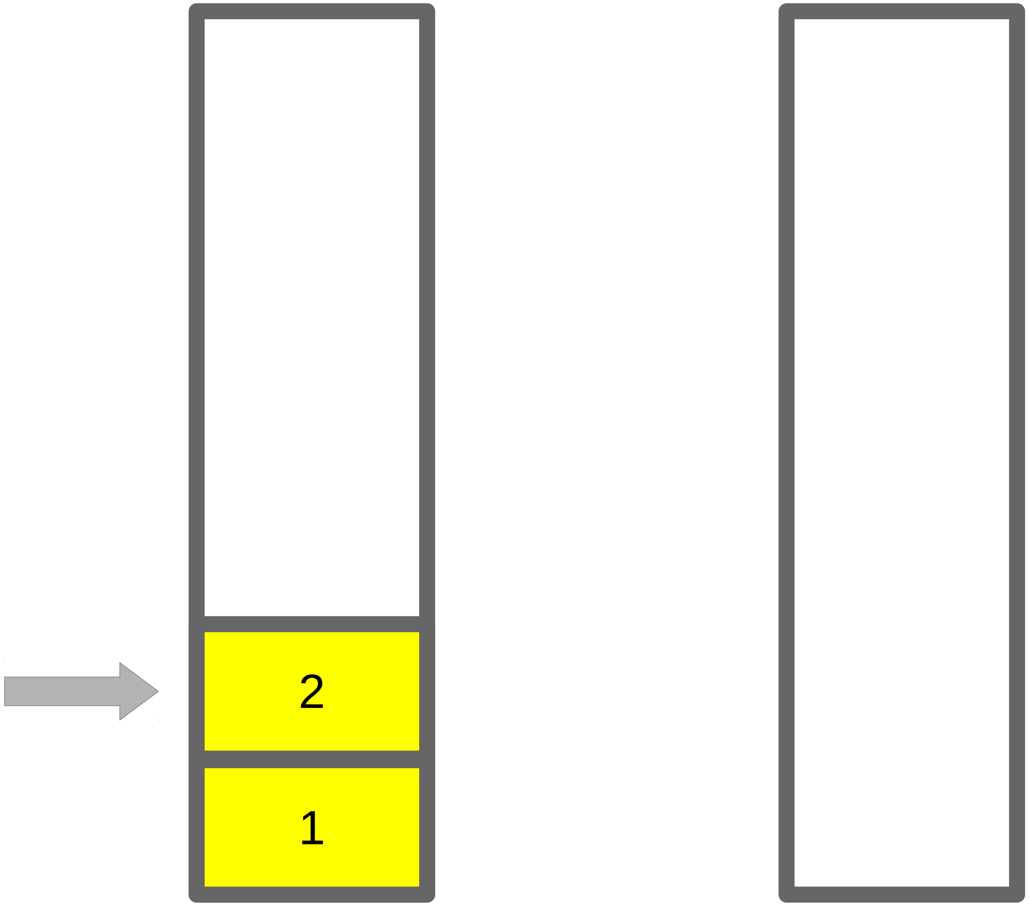

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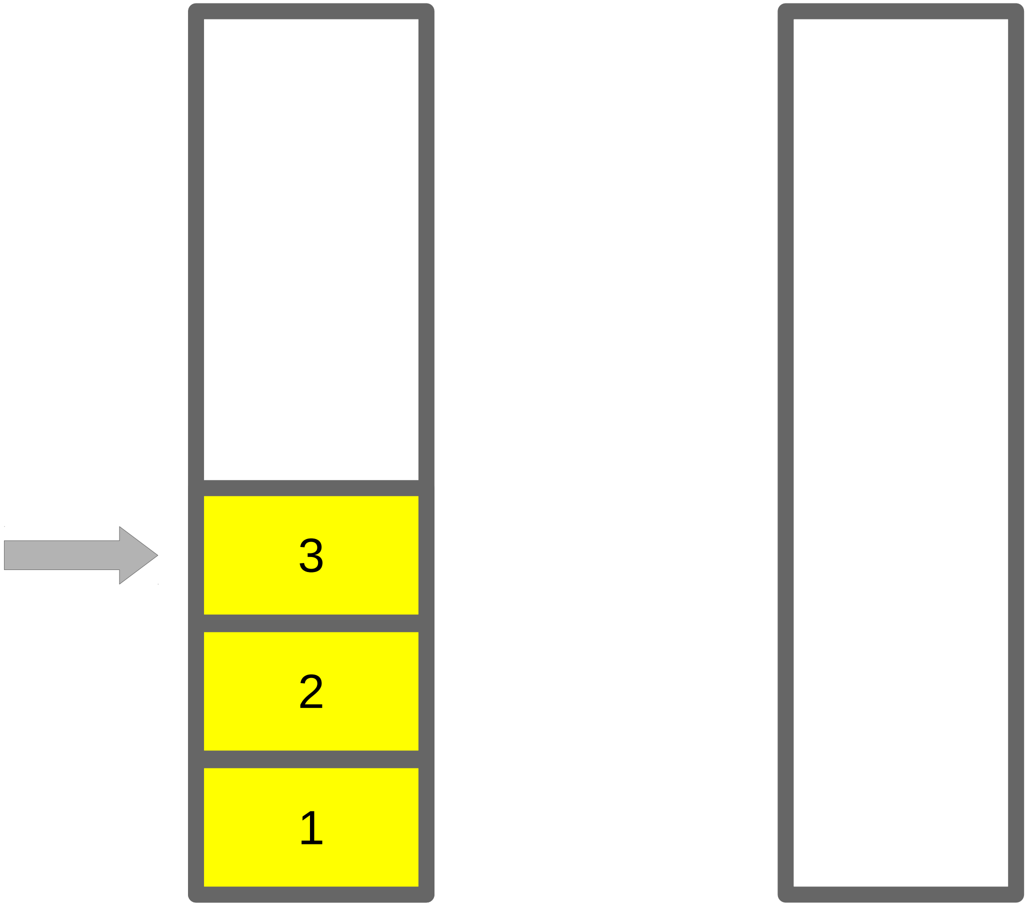

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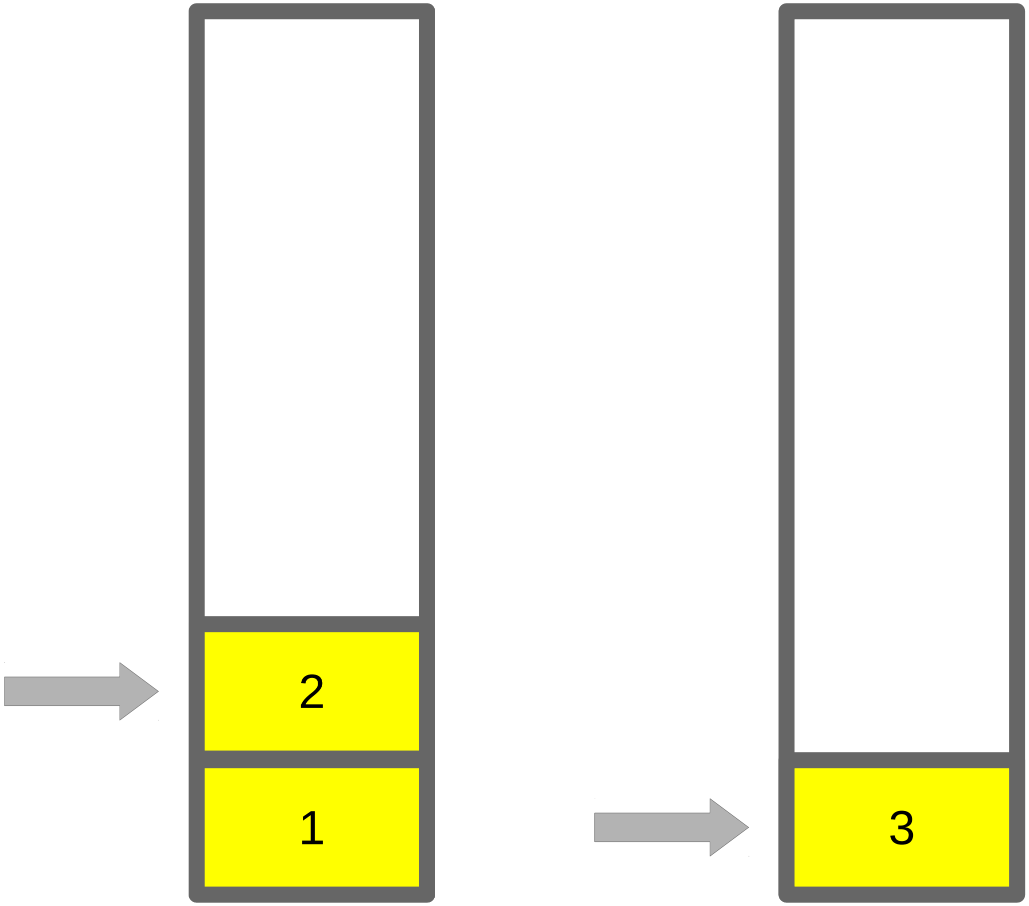

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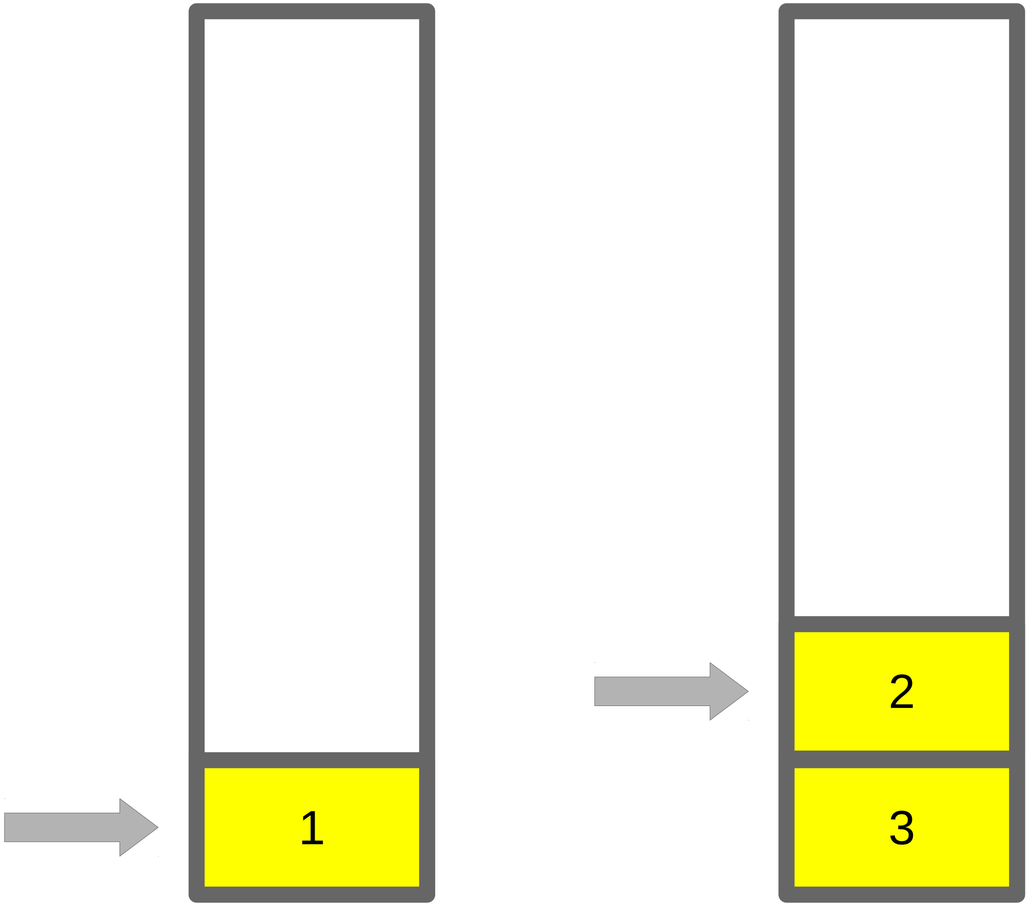

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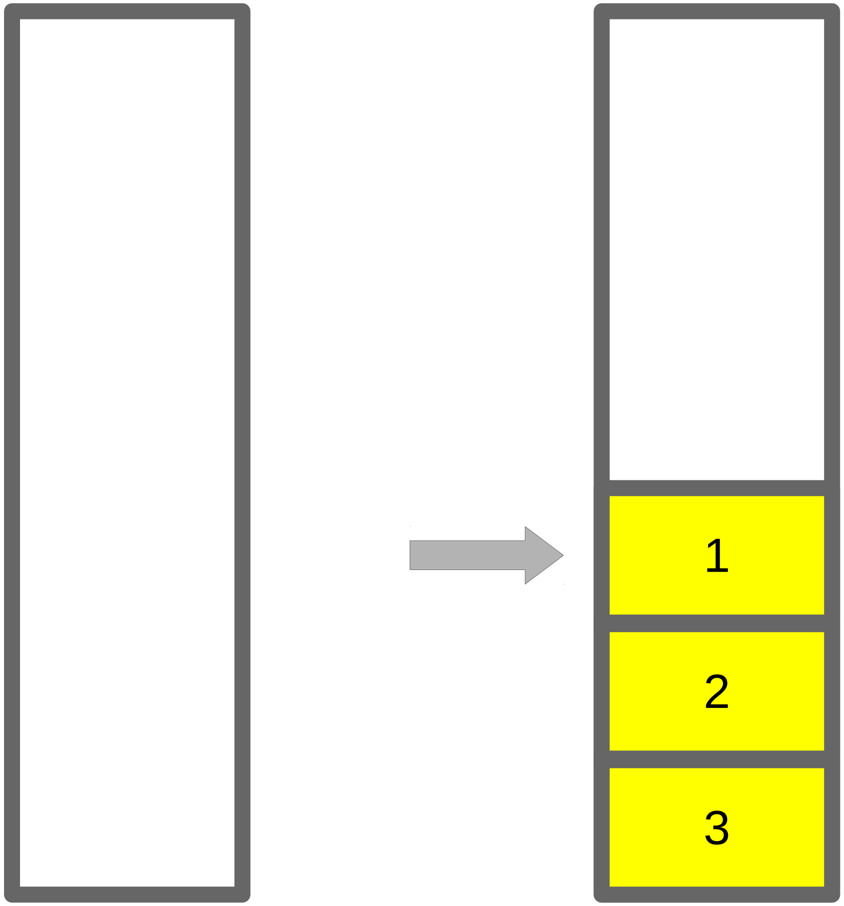

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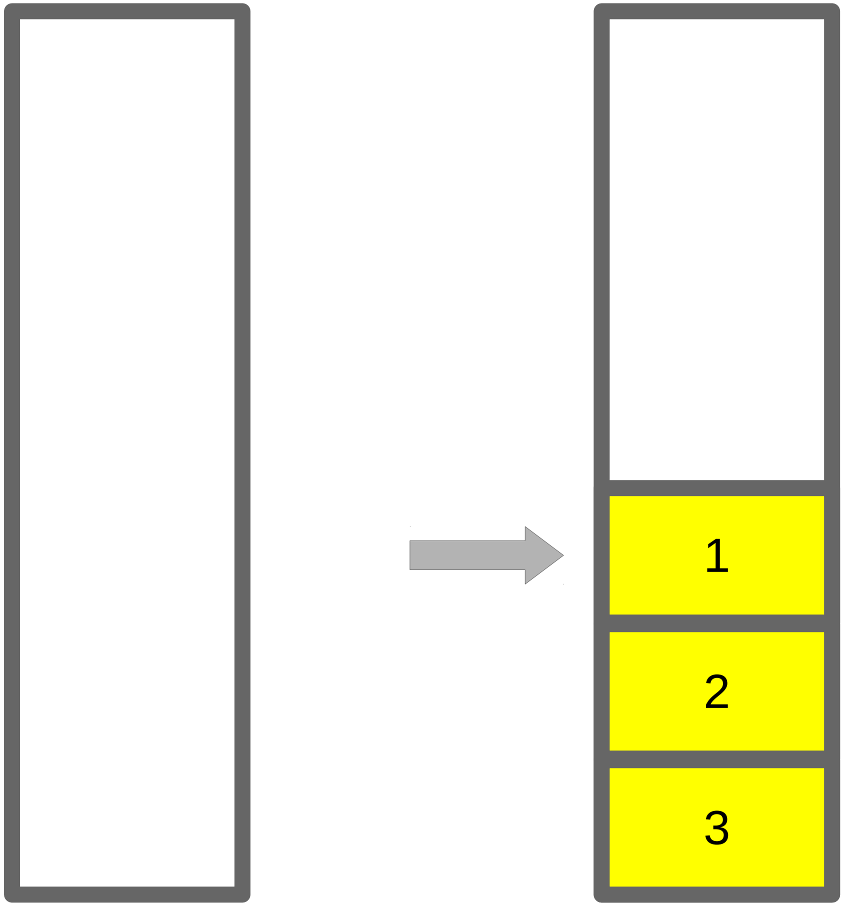

- How long does each operation take?

Amortized analysis example: aggregate analysis

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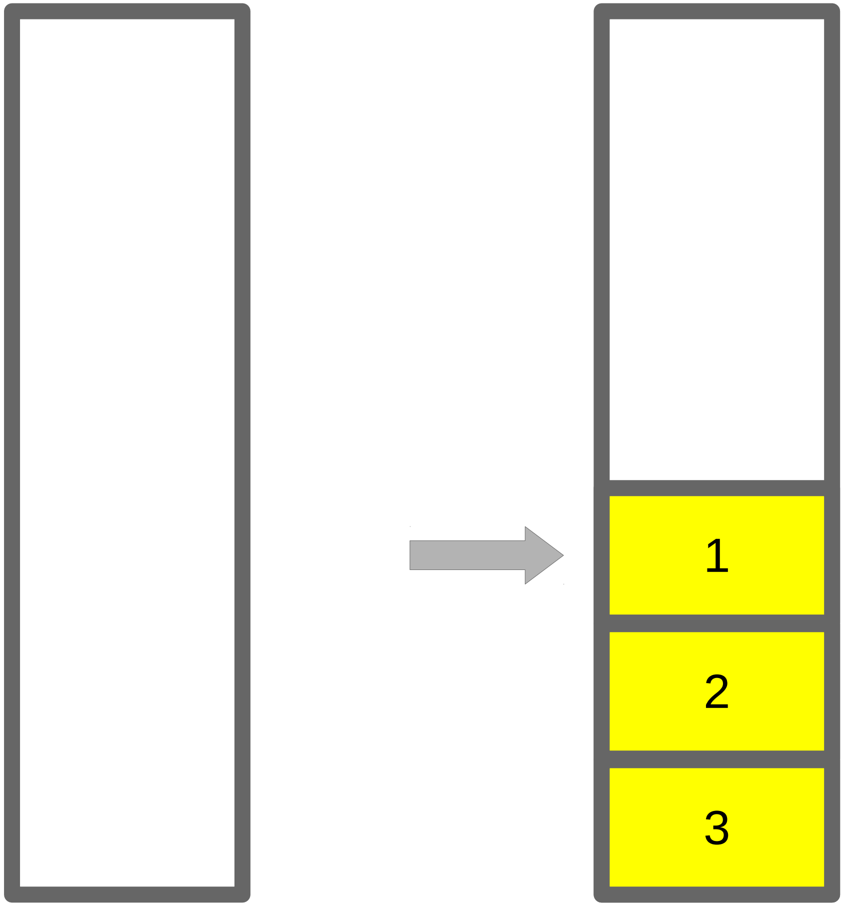
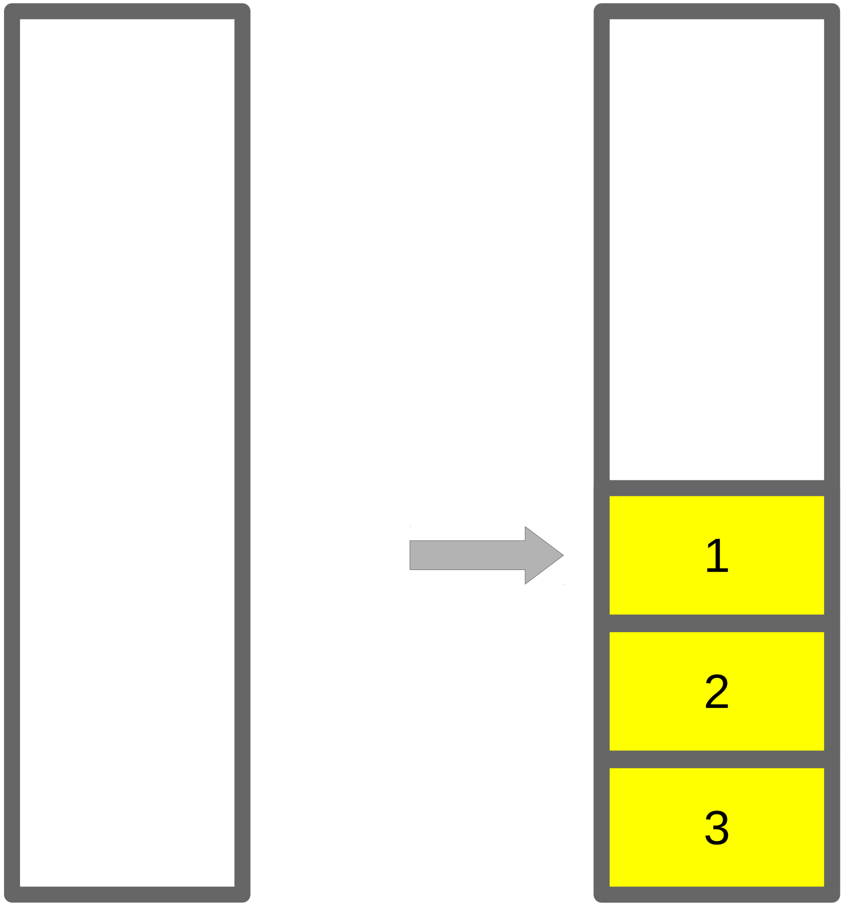


- How long does each operation take?
- How many times does each element move?

Amortized analysis example: aggregate analysis

- Implementing a queue (first in, first out) with two stacks (first in, last out)



- How long does each operation take?
- How many times does each element move?
  - Added, moved, removed

Amortized analysis example: aggregate analysis

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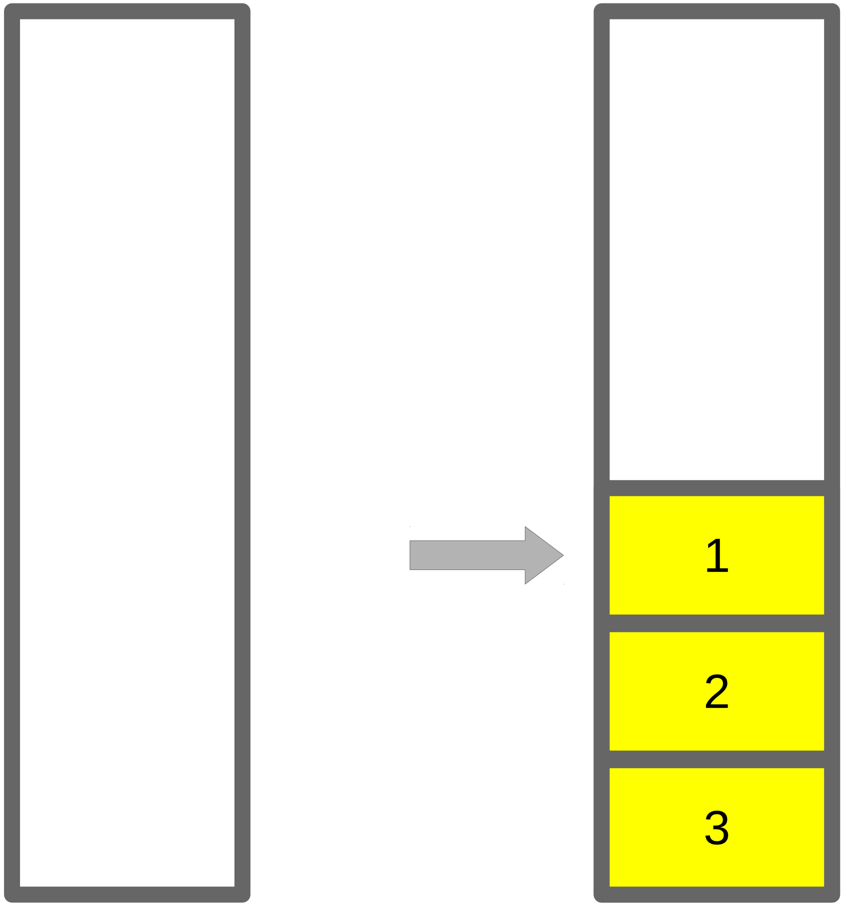

- How long does each operation take?
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  - At most 3 moves

Amortized analysis example: aggregate analysis

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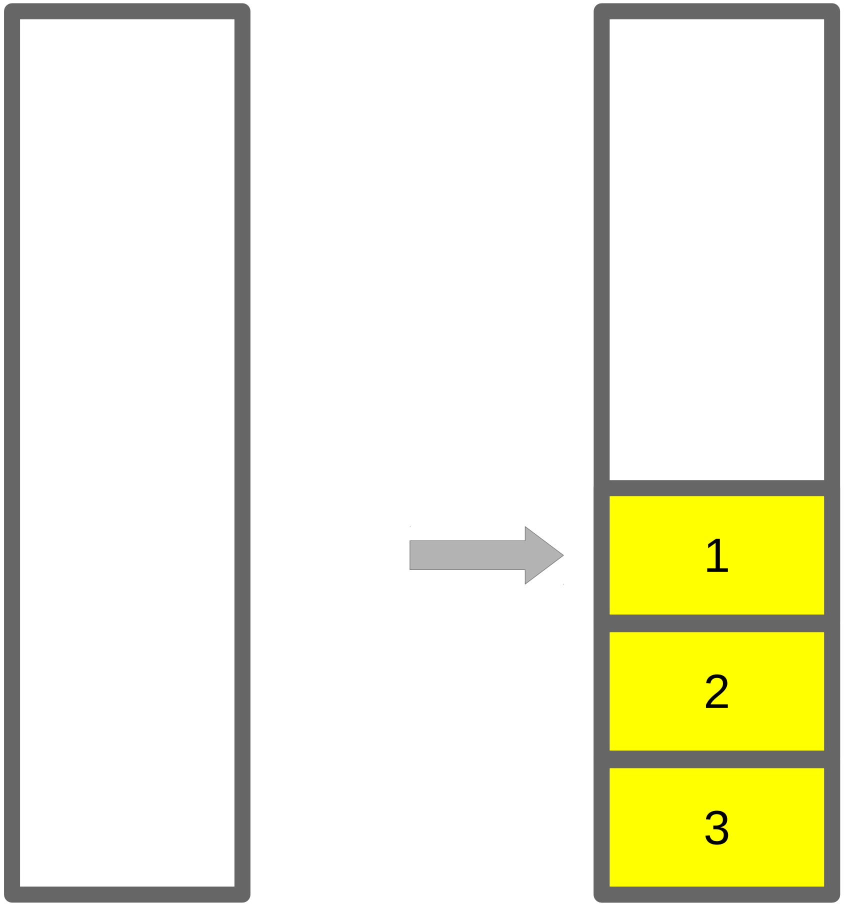

- How long does each operation take?
- How many times does each element move?
  - Added, moved, removed
  - At most 3 moves
    - O(3n) for n operations

Amortized analysis example: aggregate analysis

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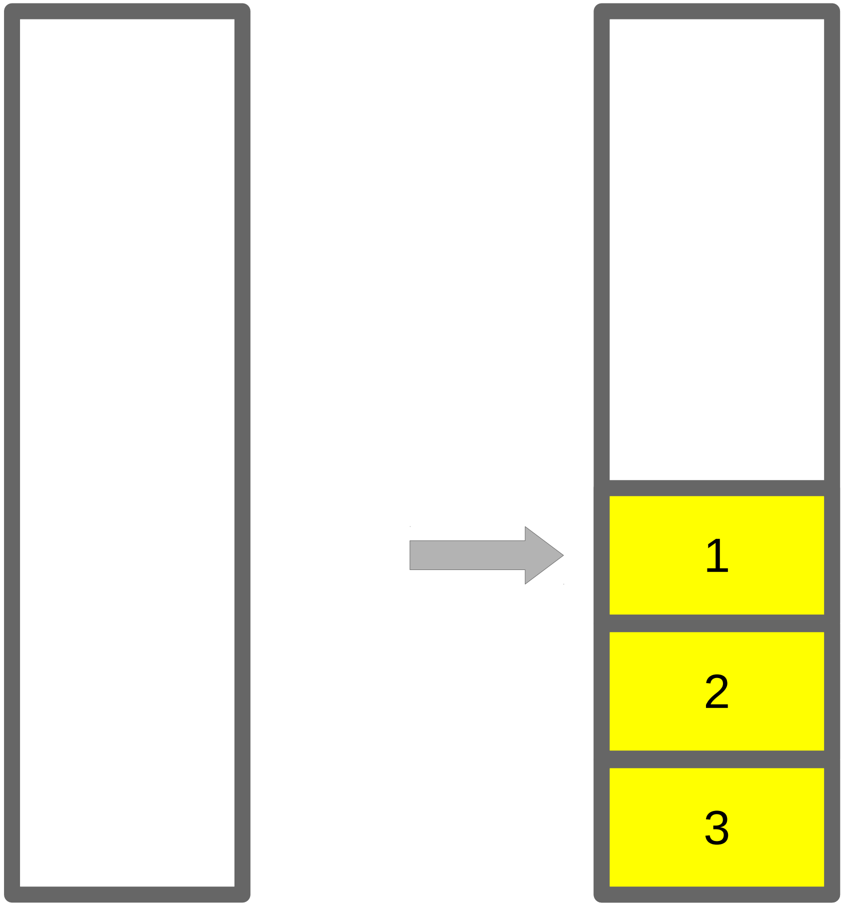

- How long does each operation take?
- How many times does each element move?
  - Added, moved, removed
  - At most 3 moves
    - O(3n) for n operations
    - Constant time for a single operation

Amortized analysis example: accounting method

Amortized analysis example: accounting method

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Amortized analysis example: accounting method

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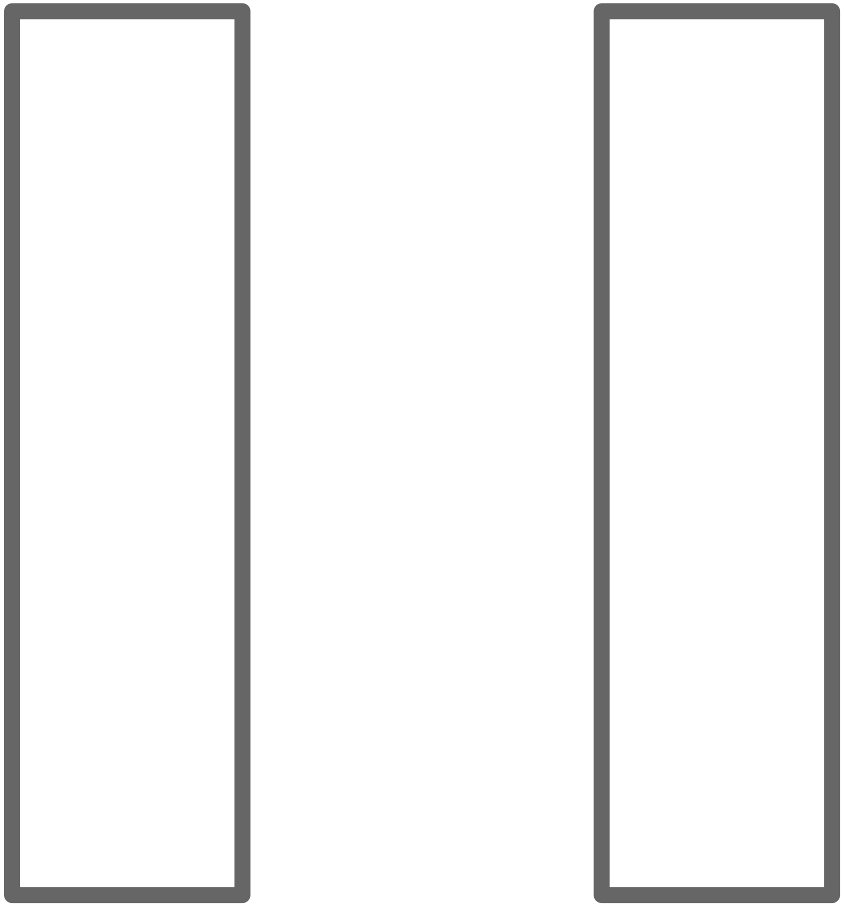

Amortized analysis example: accounting method

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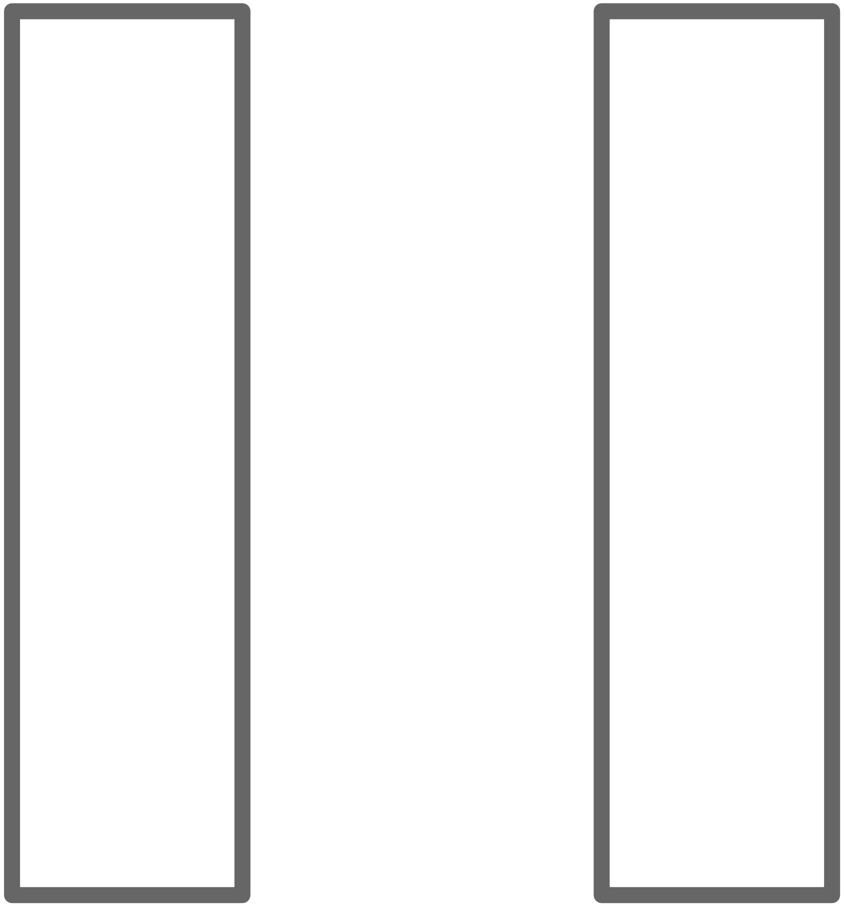

- Give each element 3 units of currency (dollars, bit coins, doge coins, etc.)

Amortized analysis example: accounting method

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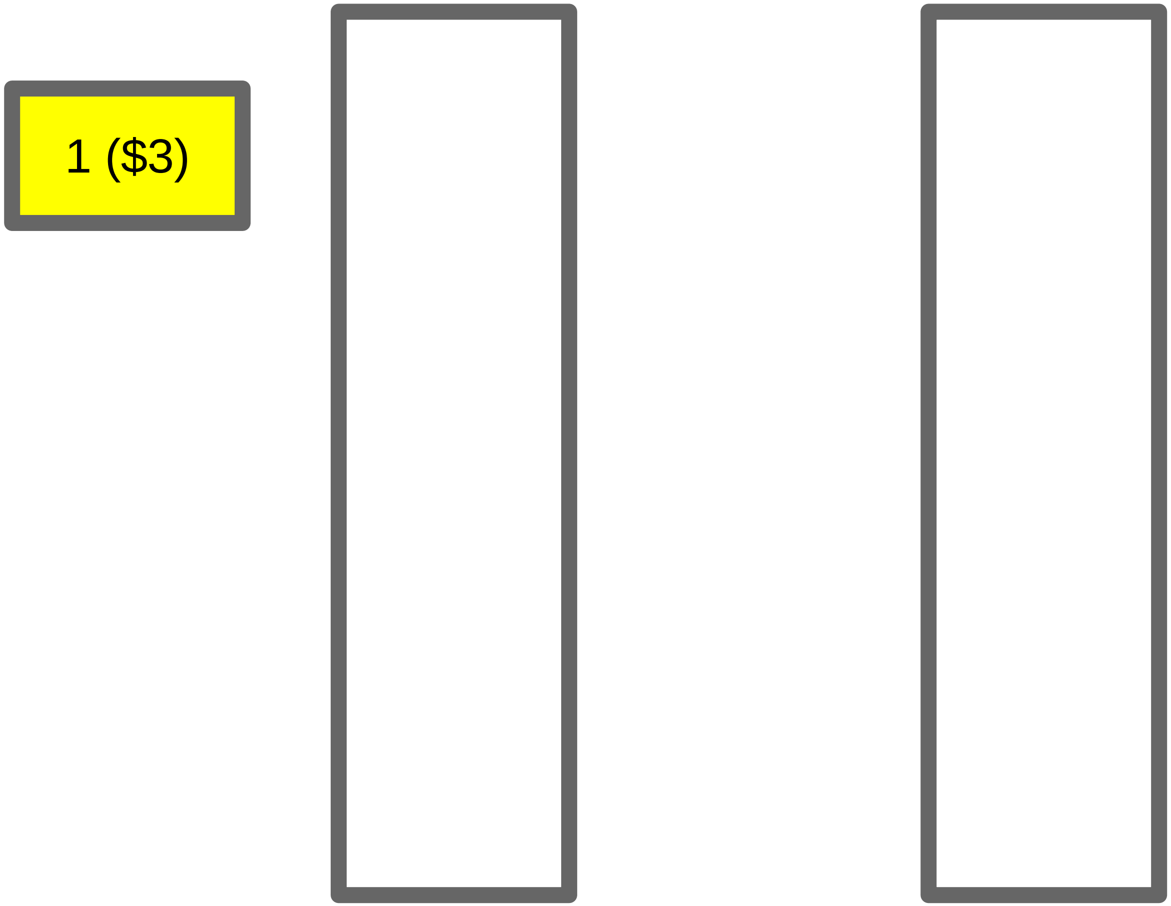

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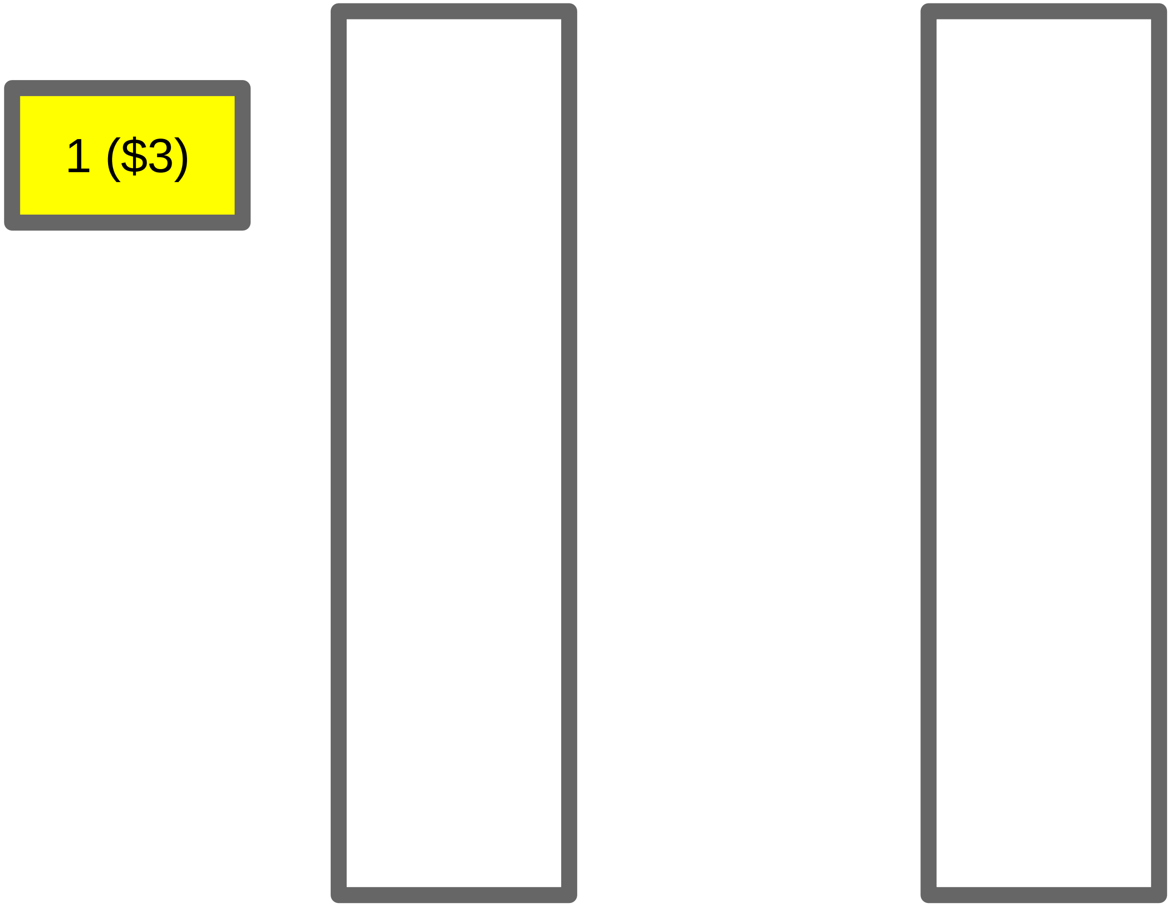

- Give each element 3 units of currency (dollars, bit coins, doge coins, etc.)
- Charge for each move

Amortized analysis example: accounting method

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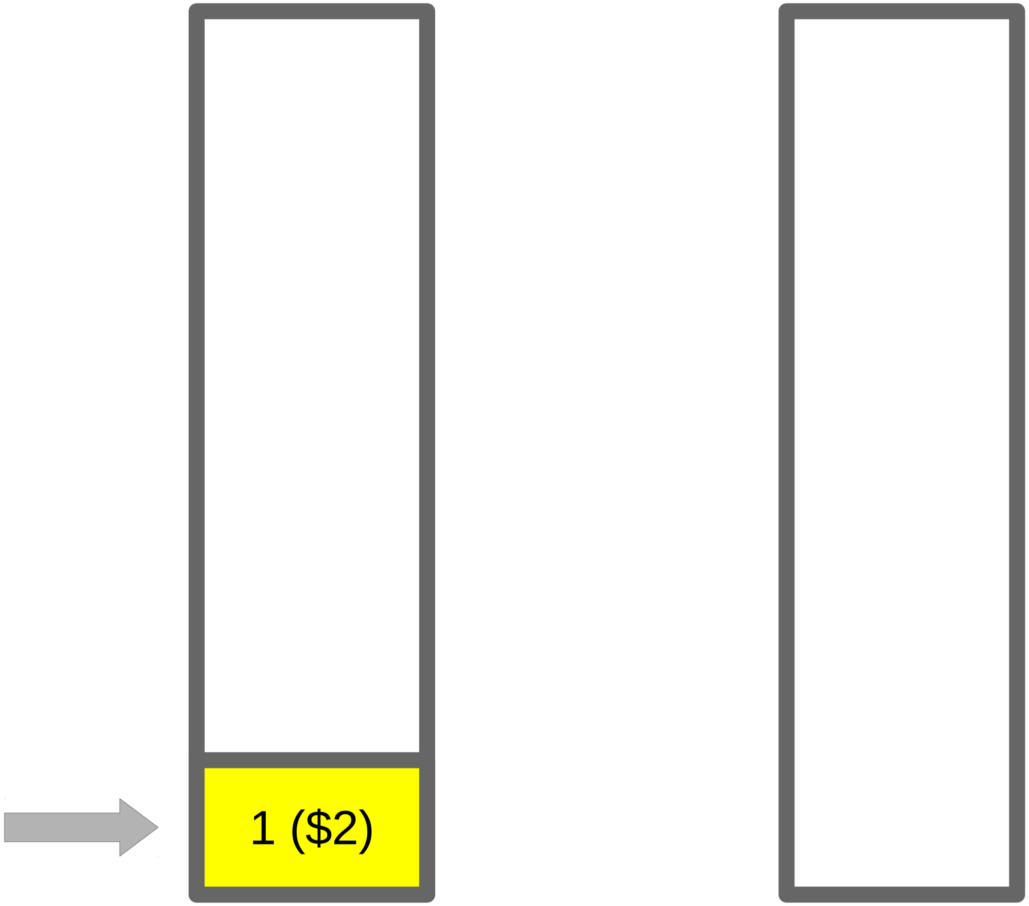

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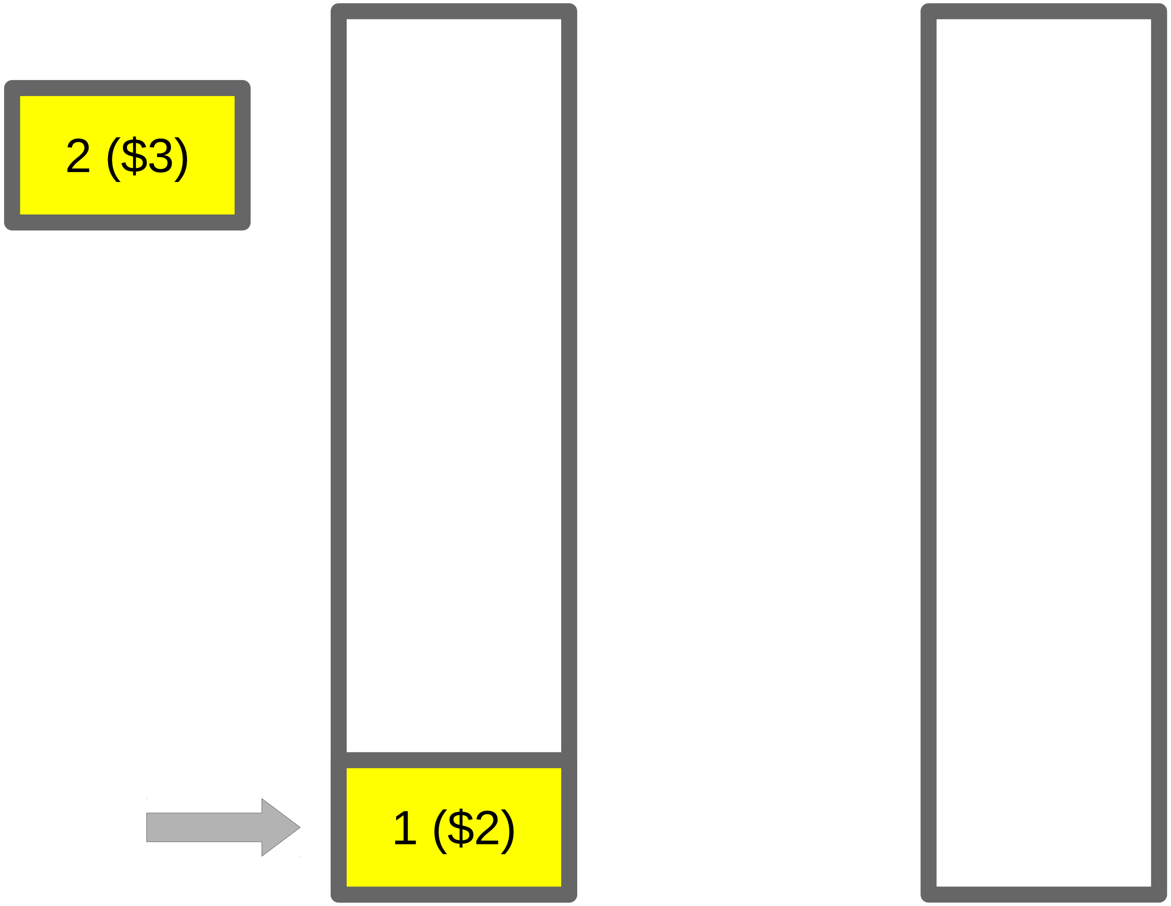

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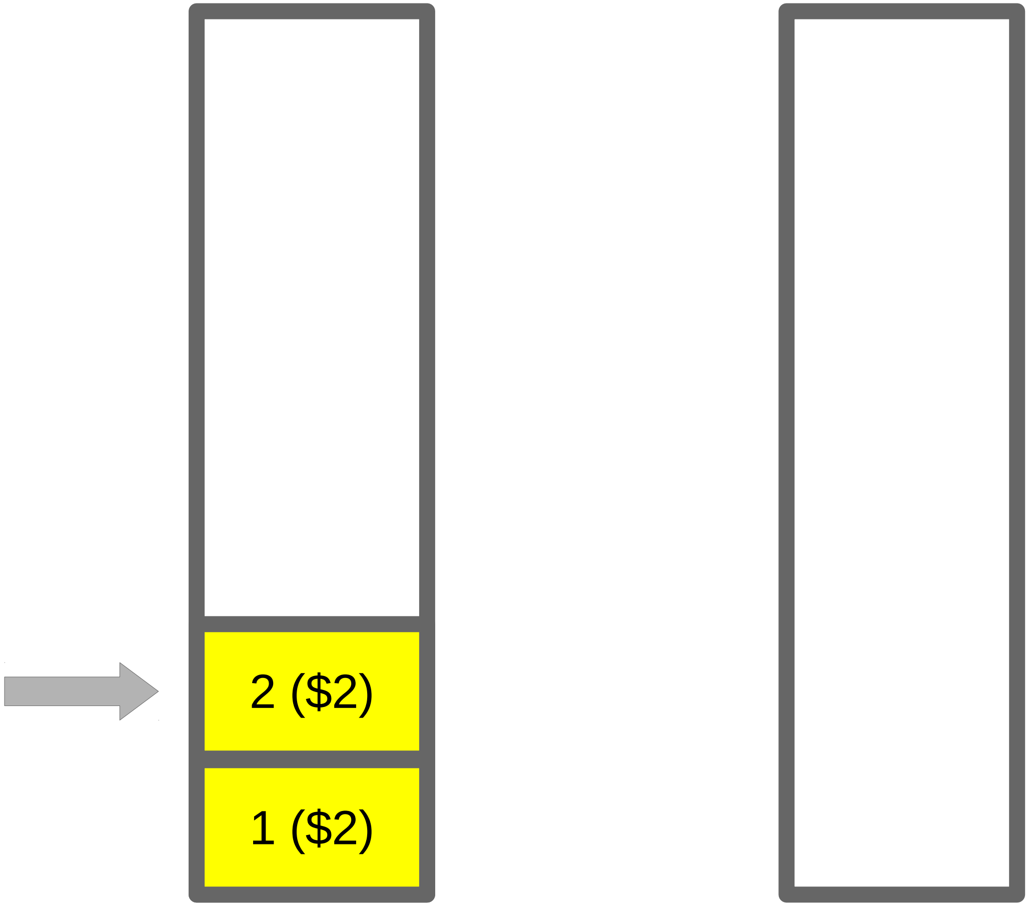

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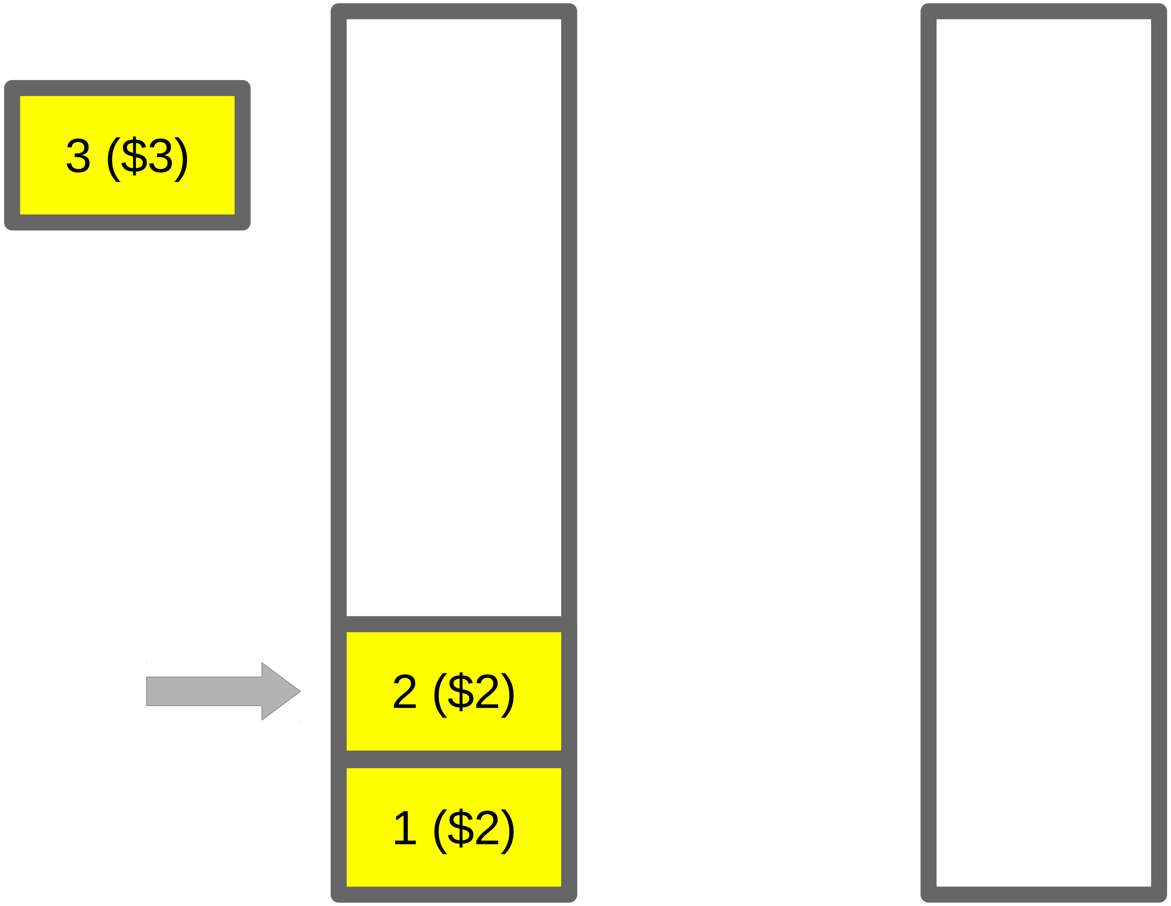

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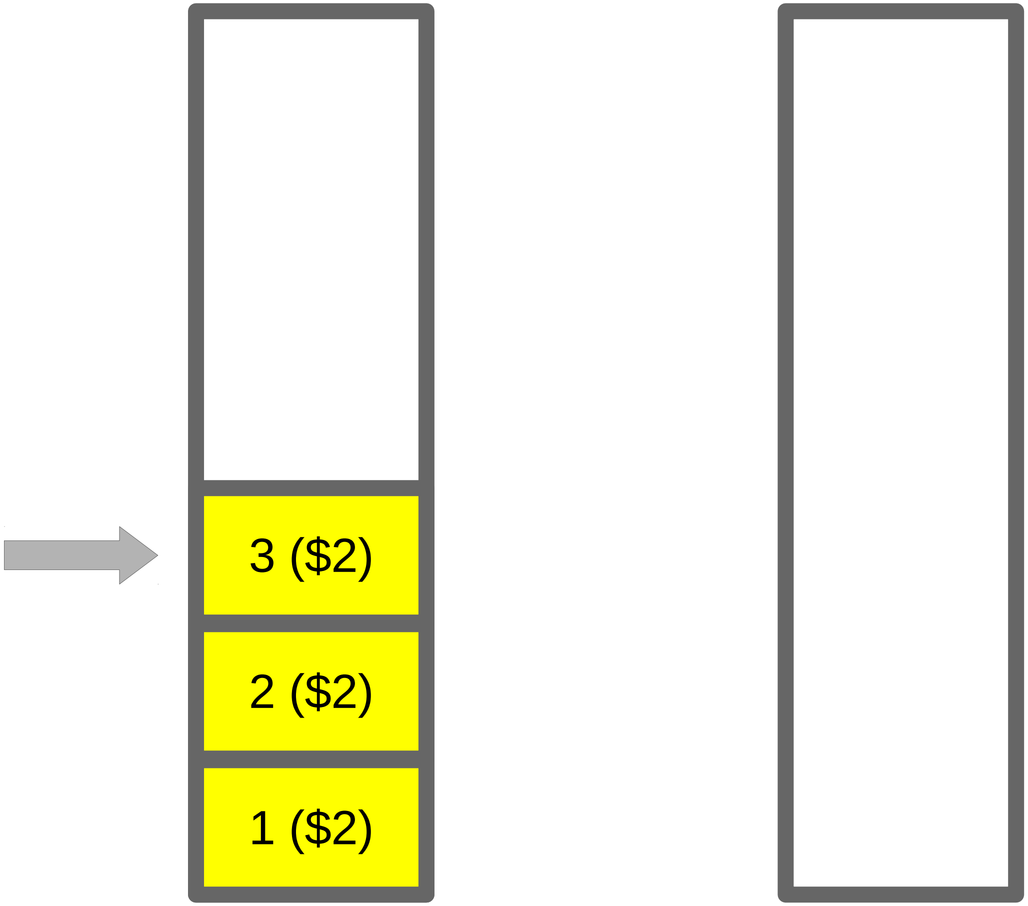

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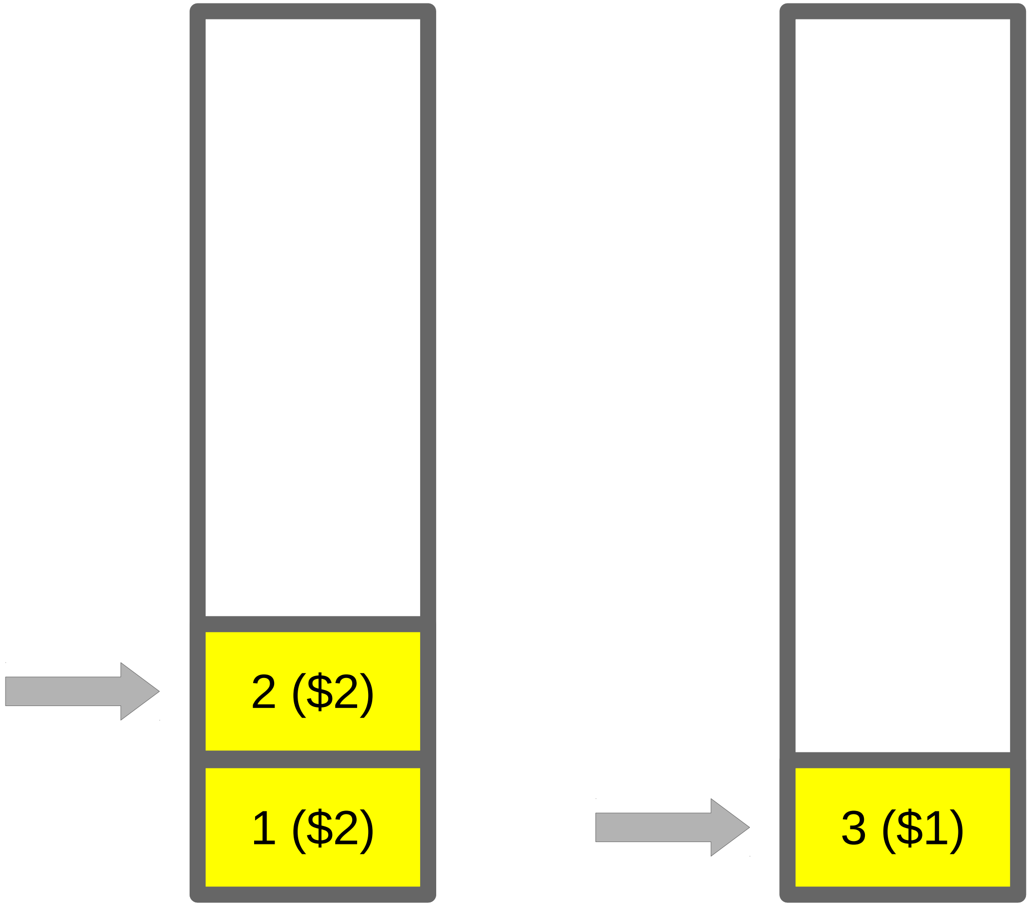

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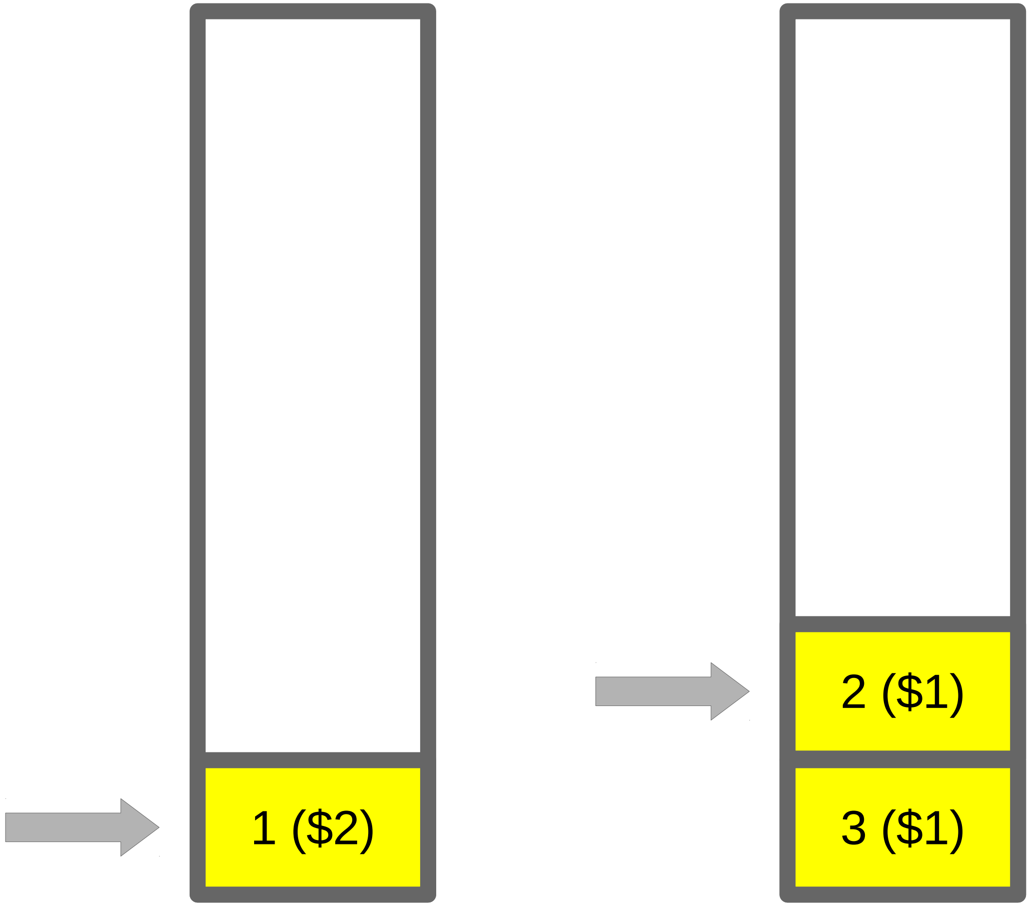

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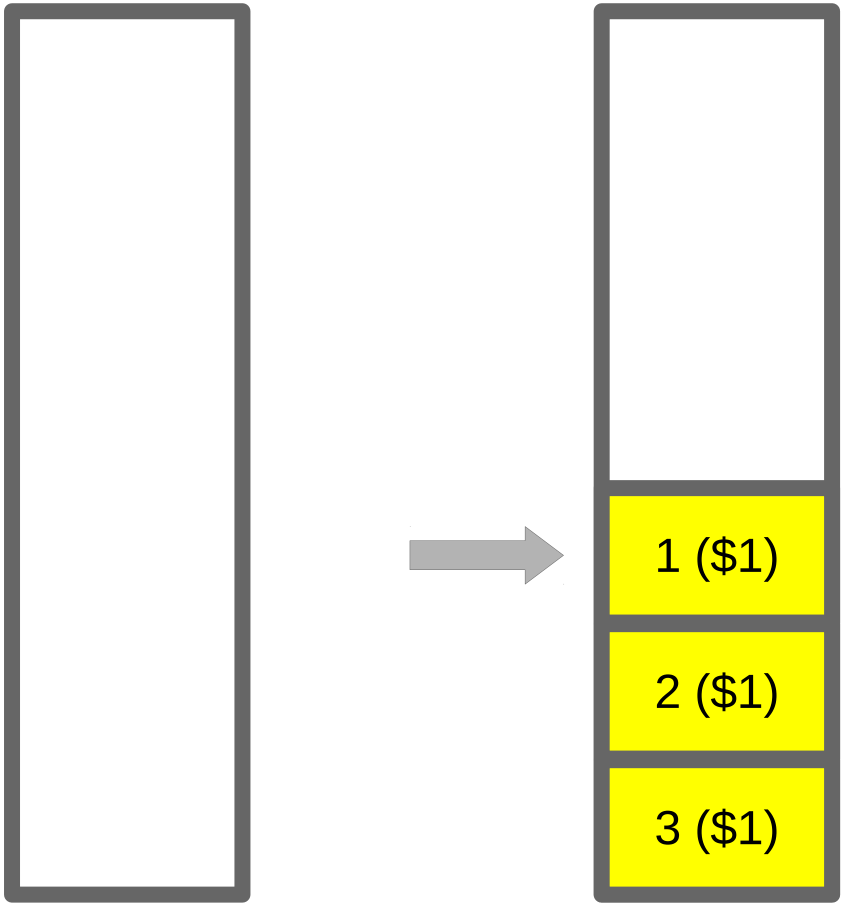

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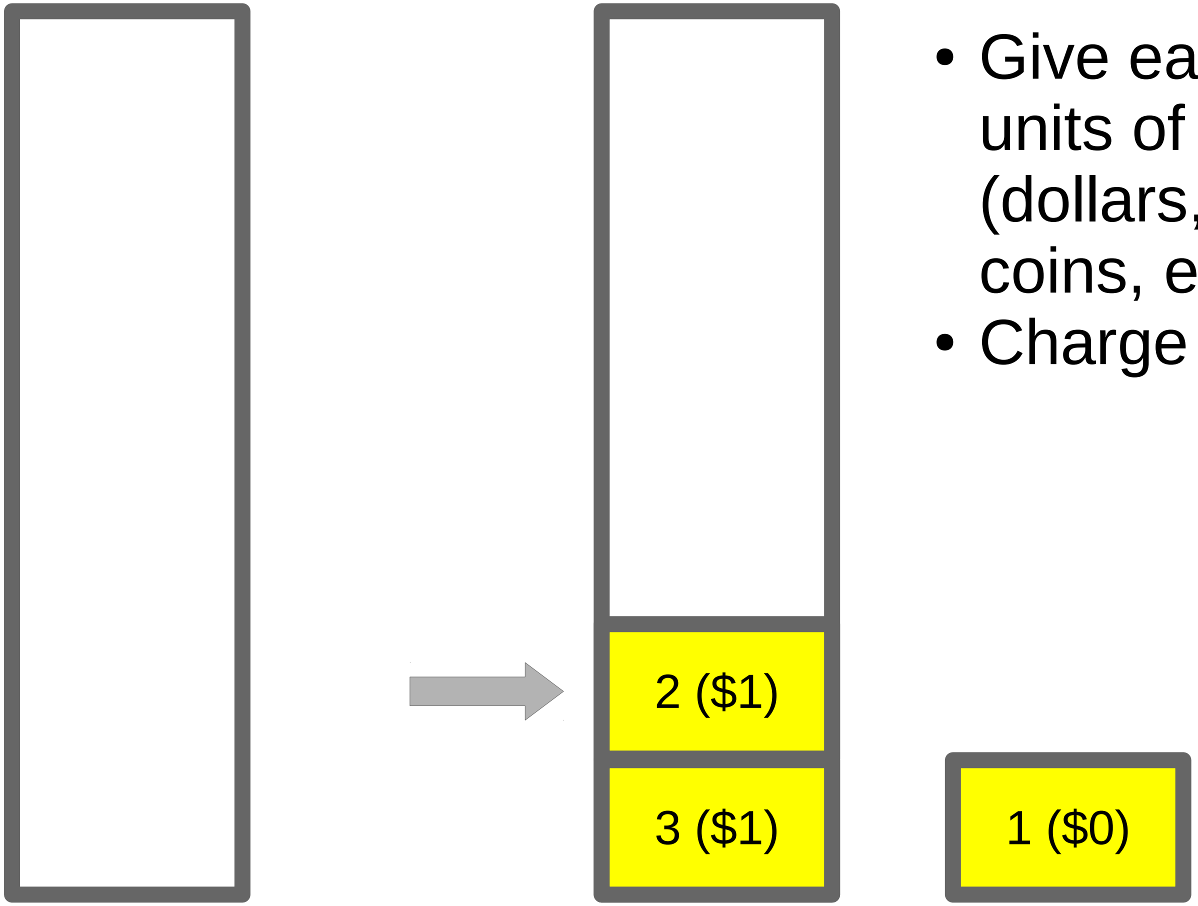

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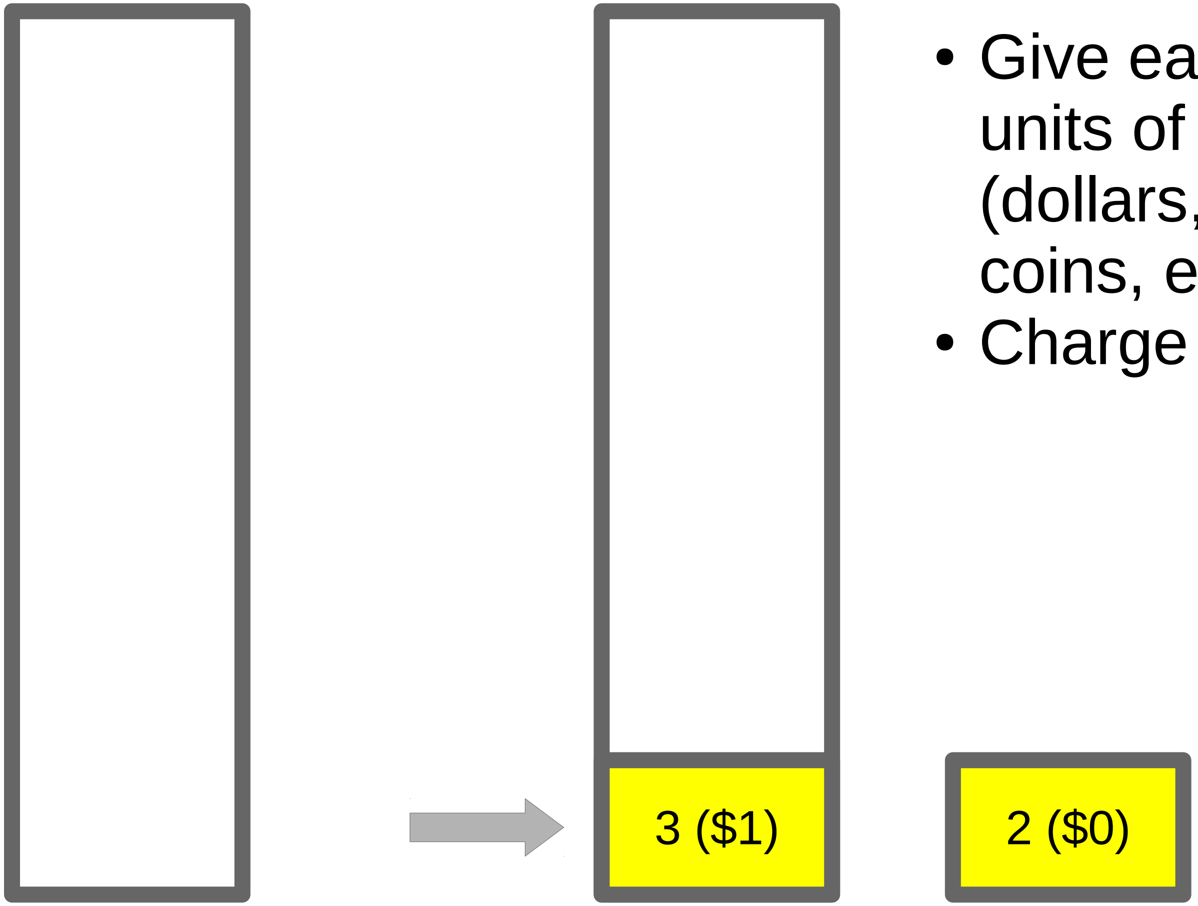

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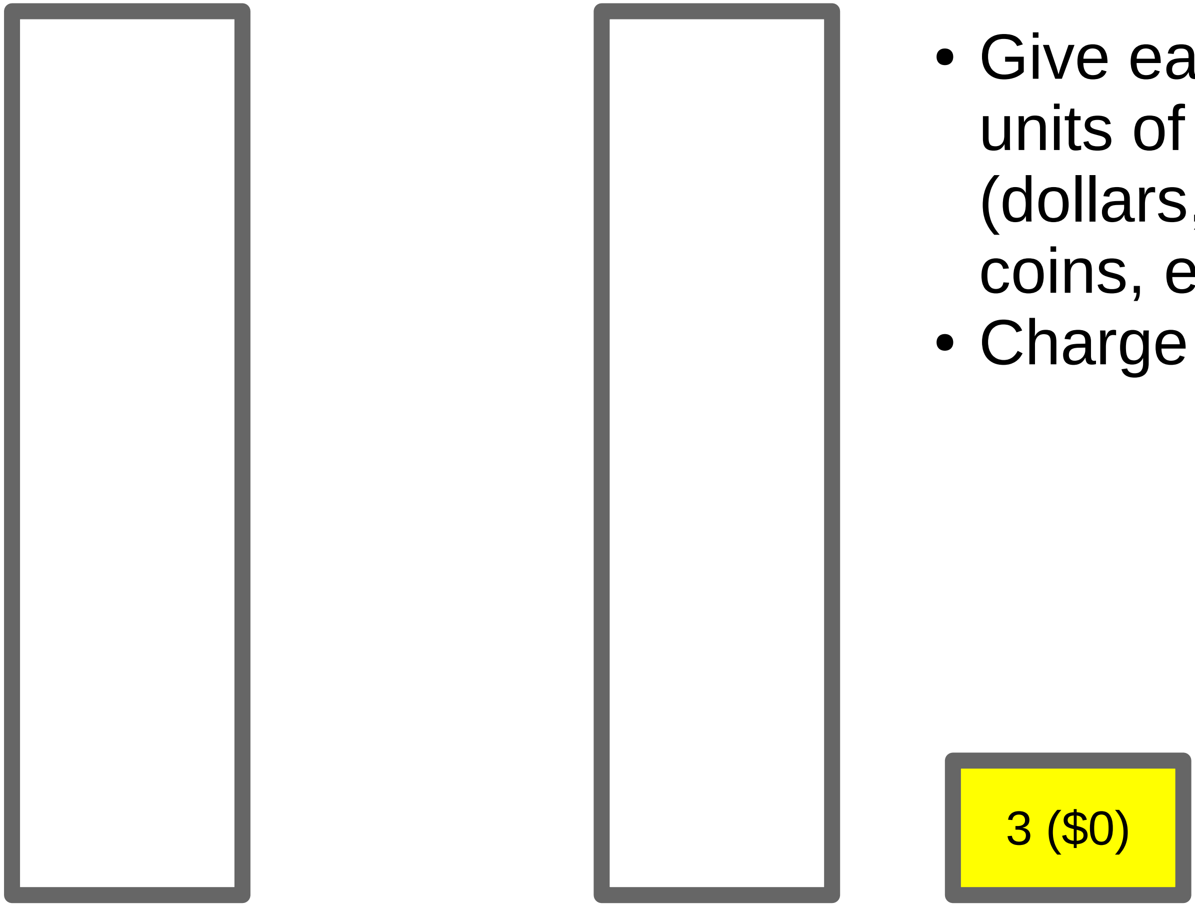

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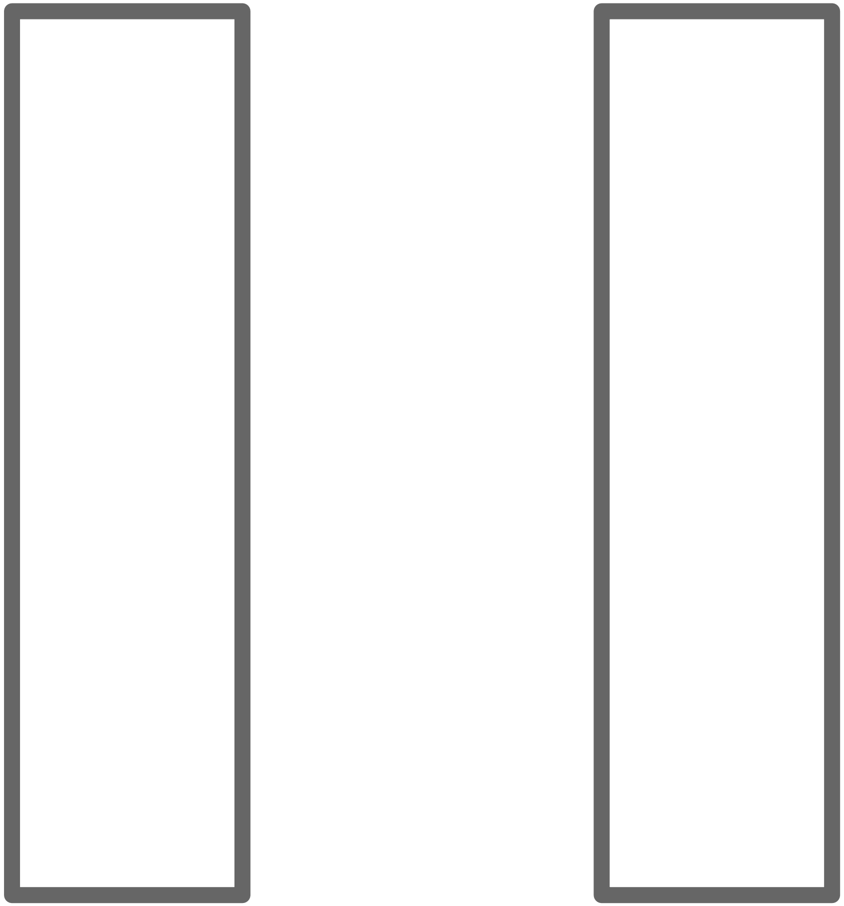
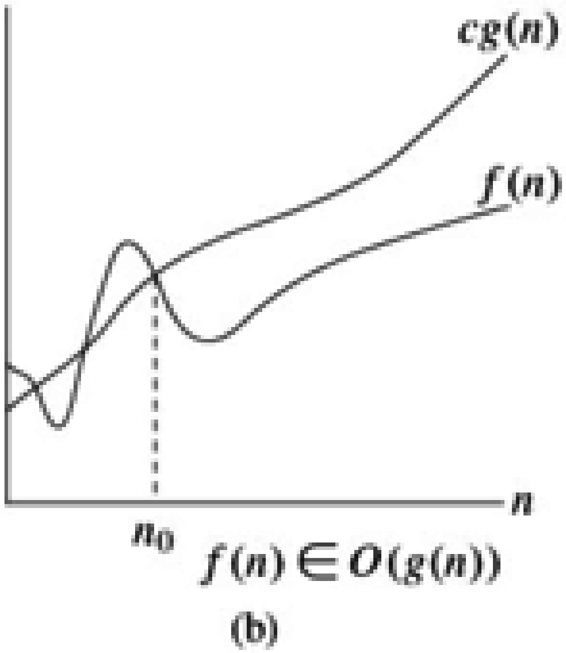


- Give each element 3 units of currency (dollars, bit coins, doge coins, etc.)
- Charge for each move
- We used 3 units of currency (constant time operations) for each element

3 ($0)

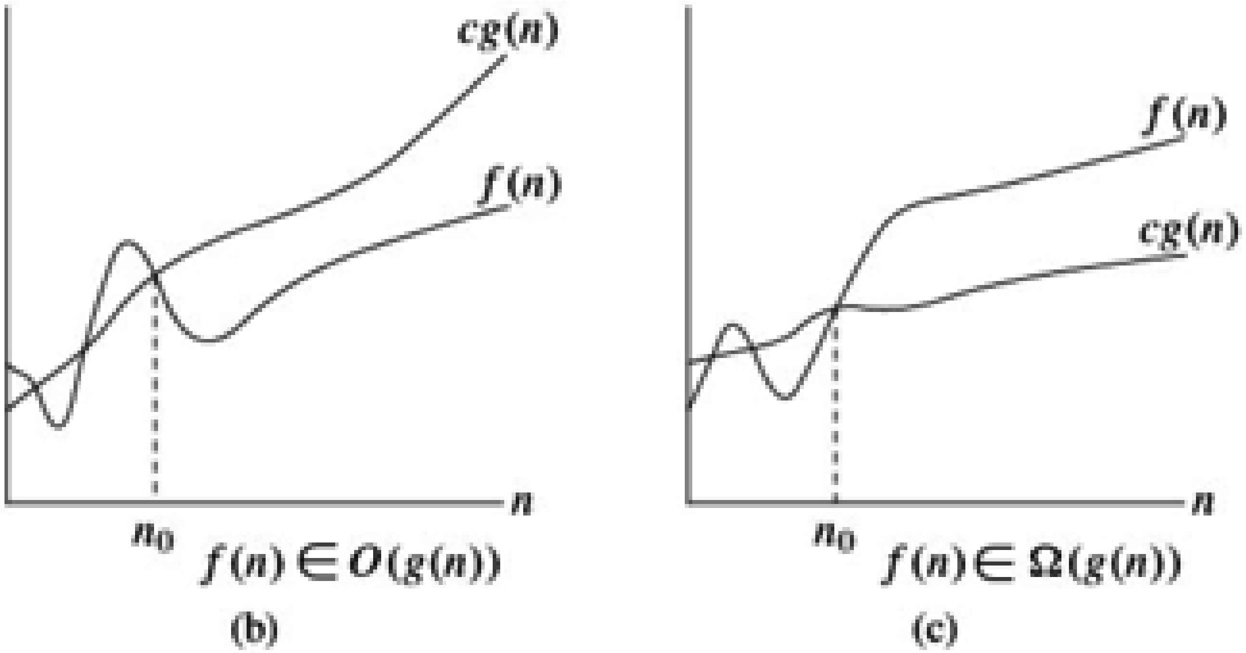
Asymptotic analysis: Big-O, Θ, and Ω notation

Asymptotic analysis: Big-O, Θ, and Ω notation



O: bounded above
given some constant

Asymptotic analysis: Big-O, Θ, and Ω notation



$f(n) \in O(g(n))$

(b)

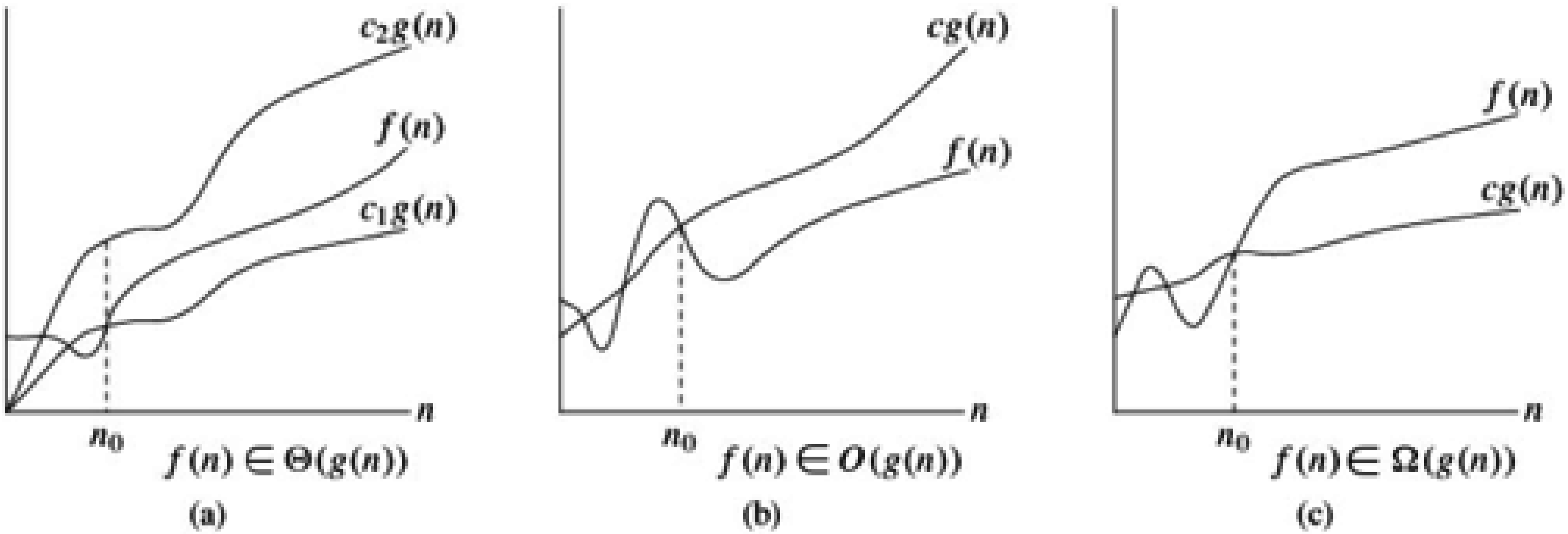
$f(n) \in \Omega(g(n))$

(c)

O: bounded above
given some constant

Ω: bounded above
given some constant

Asymptotic analysis: Big-O, Θ, and Ω notation



$f(n) \in \Theta(g(n))$

(a)

$f(n) \in O(g(n))$

(b)

$f(n) \in \Omega(g(n))$

(c)

Θ: bounded above given some constant, bounded below given some constant

O: bounded above given some constant

Ω: bounded above given some constant