

Lecture 3

- Background sequence models
 - Markov models
 - proteins
- Site models

Failure of independence assumption

Nucleotide Freqs (*C. elegans* chr. 1):

A 4575132 (.321) ; C 2559048 (.179) ; G 2555862 (.179) ; T 4582688 (.321)

dinucleotide frequencies (5' nuc to left, 3' nuc at top - e.g. obs freq of ApC is .047): (Note "symmetry"!)

	Observed				Expected (under independence)			
	A	C	G	T	A	C	G	T
A	0.135	0.047	0.051	0.088	0.103	0.057	0.057	0.103
C	0.061	0.035	0.033	0.051	0.057	0.032	0.032	0.058
G	0.063	0.034	0.034	0.047	0.057	0.032	0.032	0.057
T	0.061	0.064	0.061	0.135	0.103	0.058	0.057	0.103

	Observed / Expected			
	A	C	G	T
A	1.314	0.818	0.885	0.853
C	1.055	1.075	1.031	0.886
G	1.106	1.062	1.074	0.818
T	0.597	1.105	1.056	1.313

Conditional probability (in *C. elegans*) of a given nucleotide (top) occurring, given the preceding nucleotide (left)

	A	C	G	T
A	0.421	0.147	0.159	0.274
C	0.338	0.193	0.185	0.284
G	0.355	0.190	0.192	0.263
T	0.191	0.198	0.189	0.421

Markov models

- Conditional probabilities (as on the previous slide) can be used to define a *first-order Markov model* (or *Markov chain model*) for sequence probabilities:

$$P(s_1 s_2 s_3 \cdots s_n) \\ \equiv P(s_1) P(s_2 / s_1) P(s_3 / s_2) \cdots P(s_n / s_{n-1})$$

- Similarly, one can define an a ***order-k Markov model*** in which the probability of s_i is conditional on $s_{i-k} \dots s_{i-2} s_{i-1}$ (i.e. the k preceding residues)
- Note that the required number of parameters is exponential in k
- The ***independence model*** (which is usually good enough for us!) = the ***order-0 Markov model***

Background models for *protein* sequences

- The *independence* assumption is (usually) OK
- The *equal frequency* assumption is not

Failure of equal frequency assumption for proteins

AMINO ACID	FREQUENCY.	# SYNON CODONS.
L	.093	6
A	.075	4
S	.072	6
G	.069	4
V	.065	4
E	.063	2
K	.059	2
T	.058	4
I	.057	3
D	.053	2
R	.052	6
P	.049	4
N	.045	2
F	.041	2
Q	.040	2
Y	.032	2
M	.024	1
H	.022	2
C	.017	2
W	.013	1

Hypotheses to explain correlation between frequency and # codons

- (*Neutralist*):
 - Nucleotide sequences that encode proteins are on average close to random,
 - so amino acid freqs are proportionate to codon freqs in random DNA.
- (*Selectionist*):
 - The genetic code evolved concurrently with early proteins, and
 - is adapted so that the most useful amino acids are encoded by the most codons.
- The truth is probably some combination of these!
 - Dependence of aa composition on genomic G+C content is consistent with neutralist hypothesis

Deviations from randomness

- Compute, for each residue r , the ratio $\text{obs}_r / \text{exp}_r$ of
 - the observed frequency obs_r , to
 - the expected frequency exp_r if coding sequences were random:

$$\text{exp}_r = (\text{\#codons encoding } r) / 61$$

Amino Acid	Obs/Exp	1 st codon base	2 nd codon base	3 rd codon base	# codons
E	1.92	G	A	R	2
K	1.80	A	A	R	2
D	1.62	G	A	Y	2
M	1.46	A	T	G	1
N	1.37	A	A	Y	2
F	1.25	T	T	Y	2
Q	1.22	C	A	R	2
I	1.16	A	T	Not G	3
A	1.14	G	C	N	4
G	1.05	G	G	N	4
V	.99	G	T	N	4
Y	.98	T	A	Y	2
L	.95	C(T)	T	N	6
T	.88	A	C	N	4
W	.79	T	G	G	1
P	.74	C	C	N	4
S	.73	T(A)	C(G)	N	6
H	.67	C	A	Y	2
R	.53	C(A)	G	N	6
C	.52	T	G	Y	2

Obs/Exp Ratios

- All observed values are within factor of 2 of expected;
 - last column suggests trend towards “correcting” disparate # codons
- At codon position 1,
 - purines (*A* and *G*) predominate among over-represented amino acids,
 - pyrimidines (*C* and *T*) among under-represented amino acids.
- At codon position 2,
 - *A* and *T* predominate among over-represented amino acids,
 - *C* and *G* among under-represented amino acids.
- Hypotheses to explain *RWR* codon preference:
 - Vestige of ancestral code? (Shepherd)
 - Over-represented pattern more efficiently translated?

Site Models

- Probability models for short sequences, such as:
 - splice sites
 - translation start sites
 - promoter elements
 - protein “motifs”

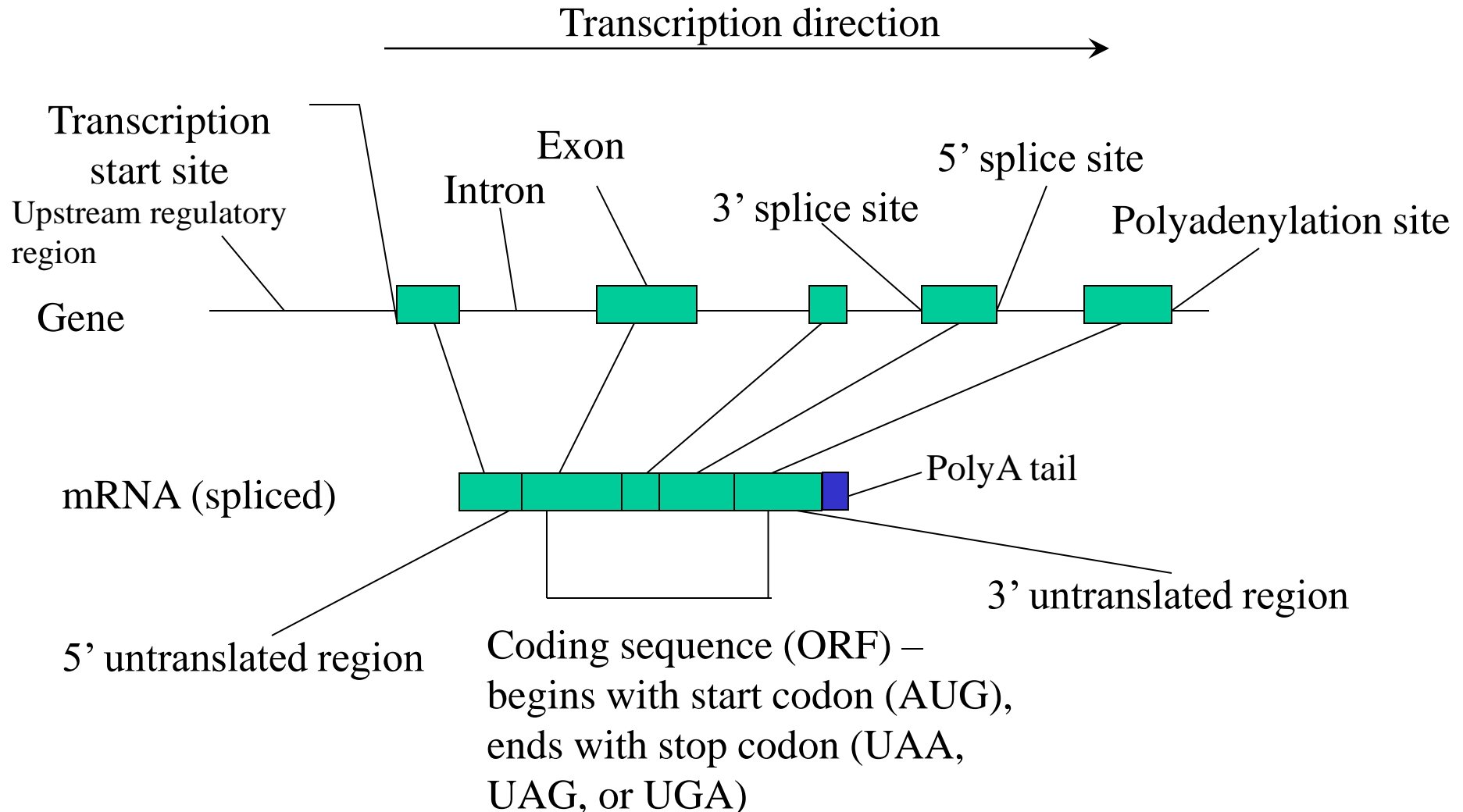
- Assumptions:
 - different examples of site can be aligned *without gaps* (indels) such that tend to have same residues in same positions
 - drop equal freq assumption: allow *position-specific freqs*
 - retain *independence* assumption (for now)

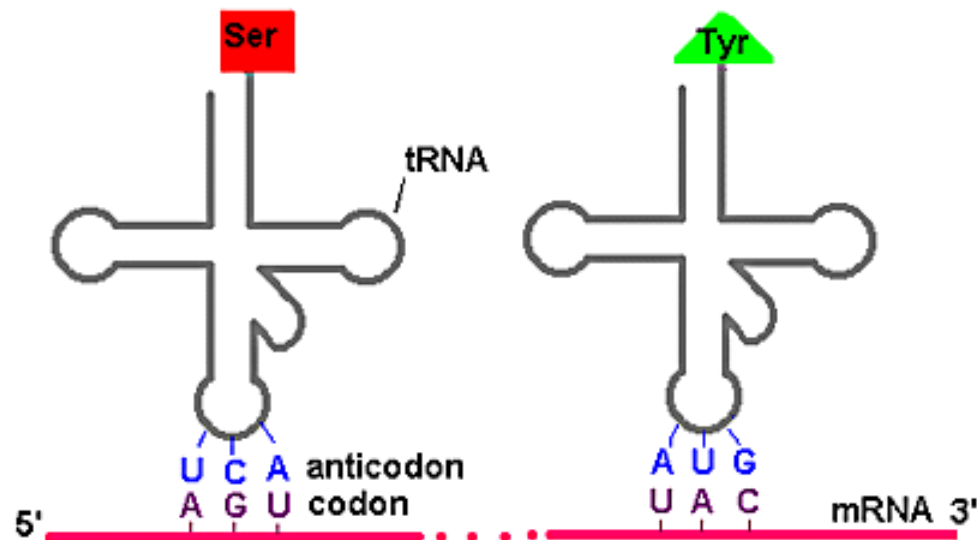
- Applies to short segments (< 30 residues) where
 - precise residue spacing is structurally or functionally important, and
 - certain positions are highly conserved
- Examples:
 - DNA/RNA sequences binding a single protein or RNA molecule
 - Protein internal regions structurally constrained due to folding requirements; or
 - protein surface regions constrained because bind certain ligands

Construction of Site Models

- Collect examples of site
- Align (without gaps)
- Count occurrences of residues at each position
- Convert to frequencies

(Protein-coding) Gene Structure in Eukaryotes





2nd base in codon

		U	C	A	G		
1st base in codon	U	Phe Phe Leu Leu	Ser Ser Ser Ser	Tyr Tyr STOP STOP	Cys Cys STOP Trp	U C A G	3rd base in codon
	C	Leu Leu Leu Leu	Pro Pro Pro Pro	His His Gln Gln	Arg Arg Arg Arg	U C A G	
	A	Ile Ile Ile Met	Thr Thr Thr Thr	Asn Asn Lys Lys	Ser Ser Arg Arg	U C A G	
	G	Val Val Val Val	Ala Ala Ala Ala	Asp Asp Glu Glu	Gly Gly Gly Gly	U C A G	

The Genetic Code

Codon Usage

- In most organisms, the codons for an amino acid are not used with equal frequency – “synonymous codon bias”.
- For many organisms this may reflect differences in translational efficiency & accuracy: more highly expressed genes have stronger biases.
- For mammals codon usage mainly reflects the GC content of the region in which the gene is found; reasons for GC content variation unknown.
- *Even though we don't fully understand the biological basis for this bias, it provides a powerful tool for gene identification!*

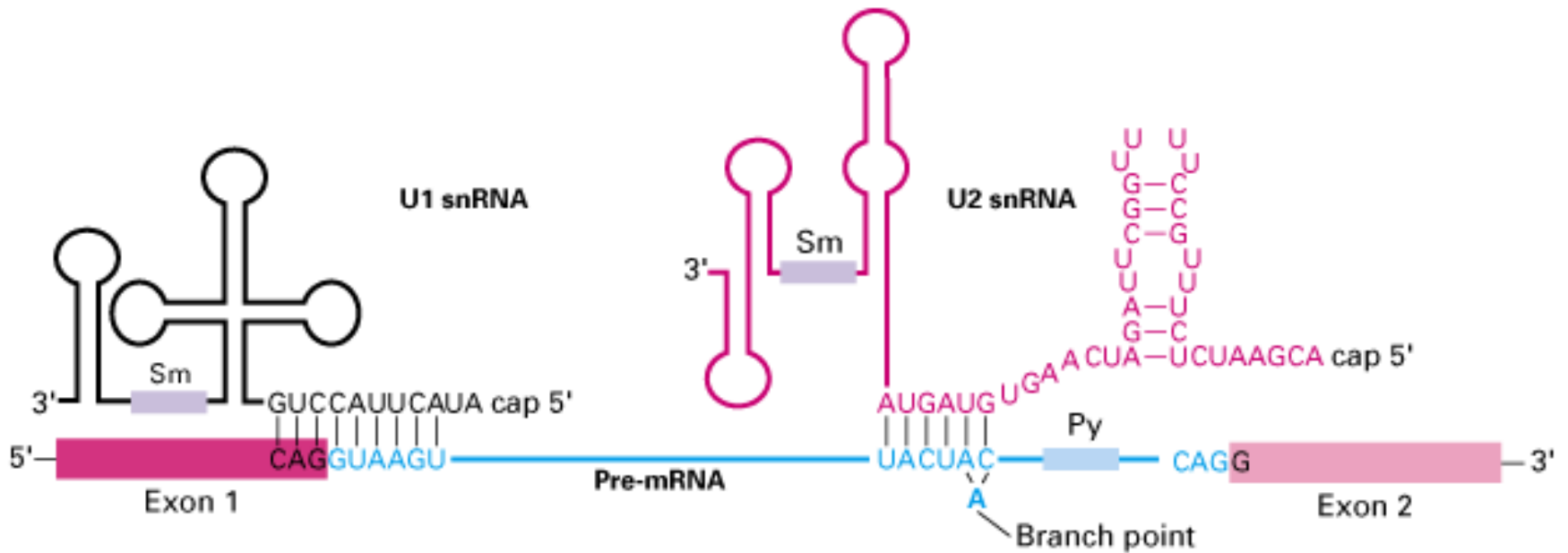
Phe	[171 UUU	\	AAA 0	Ser	[147 UCU	/	AGA 10	Tyr	[124 UAU	\	AUA 1	Cys	[99 UGU	\	ACA 0
		203 UUC		GAA 14			172 UCC		GGA 0			158 UAC		GUA 11			119 UGC		GCA 30
Leu	[73 UUA	-	UAA 8	stop	-	118 UCA	-	UGA 5	stop	-	0 UAA	-	UUA 0	stop	-	0 UGA	-	UCA 0
		125 UUG	-	CAA 6			45 UCG	-	CGA 4			0 UAG	-	CUA 0			Trp	-	122 UGG

Leu	[127 CUU	/	AAG 13	Pro	[175 CCU	/	AGG 11	His	[104 CAU	\	AUG 0	Arg	[47 CGU	/	ACG 9		
		187 CUC		GAG 0			197 CCC		GGG 0			147 CAC	\	GUG 12			107 CGC		GCG 0		
		69 CUA	-	UAG 2			170 CCA	-	UGG 10			Gln	[121 CAA			-	UUG 11	63 CGA	-	UCG 7
		392 CUG	-	CAG 6			69 CCG	-	CGG 4					343 CAG			-	CUG 21	115 CGG	-	CCG 5

Ile	[165 AUU	/	AAU 13	Thr	[131 ACU	/	AGU 8	Asn	[174 AAU	\	AUU 1	Ser	[121 AGU	\	ACU 0
		218 AUC		GAU 1			192 ACC		GGU 0			199 AAC	\	GUU 33			191 AGC	\	GCU 7
Met	-	71 AUA	-	UAU 5	Lys	[150 ACA	-	UGU 10	Lys	[248 AAA	-	UUU 16	Arg	[113 AGA	-	UCU 5
		221 AUG	-	CAU 17			63 ACG	-	CGU 7			331 AAG	-	CUU 22			110 AGG	-	CCU 4

Val	[111 GUU	/	AAC 20	Ala	[185 GCU	/	AGC 25	Asp	[230 GAU	\	AUC 0	Gly	[112 GGU	\	ACC 0		
		146 GUC		GAC 0			282 GCC		GGC 0			262 GAC	\	GUC 10			230 GGC	\	GCC 11		
		72 GUA	-	UAC 5			160 GCA	-	UGC 10			Glu	[301 GAA			-	UUC 14	168 GGA	-	UCC 5
		288 GUG	-	CAC 19			74 GCG	-	CGC 5					404 GAG			-	CUC 8	160 GGG	-	CCC 8

Figure 34 The human genetic code and associated tRNA genes. For each of the 64 codons, we show: the corresponding amino acid; the observed frequency of the codon per 10,000 codons; the codon; predicted wobble pairing to a tRNA anticodon (black lines); an unmodified tRNA anticodon sequence; and the number of tRNA genes found with this anticodon. For example, phenylalanine is encoded by UUU or UUC; UUC is seen more frequently, 203 to 171 occurrences per 10,000 total codons; both codons are expected to be decoded by a single tRNA anticodon type, GAA, using a G/U wobble; and there are 14 tRNA genes found with this anticodon. The modified anticodon sequence in the mature tRNA is not shown, even where post-transcriptional modifications can be confidently predicted (for example, when an A is used to decode a U/C third position, the A is almost certainly an inosine in the mature tRNA). The Figure also does not show the number of distinct tRNA species (such as distinct sequence families) for each anticodon; often there is more than one species for each anticodon.



from http://departments.oxy.edu/biology/Stillman/bi221/111300/processing_of_hnrnas.htm

(Jonathon Stillman, Grace Fisher-Adams)

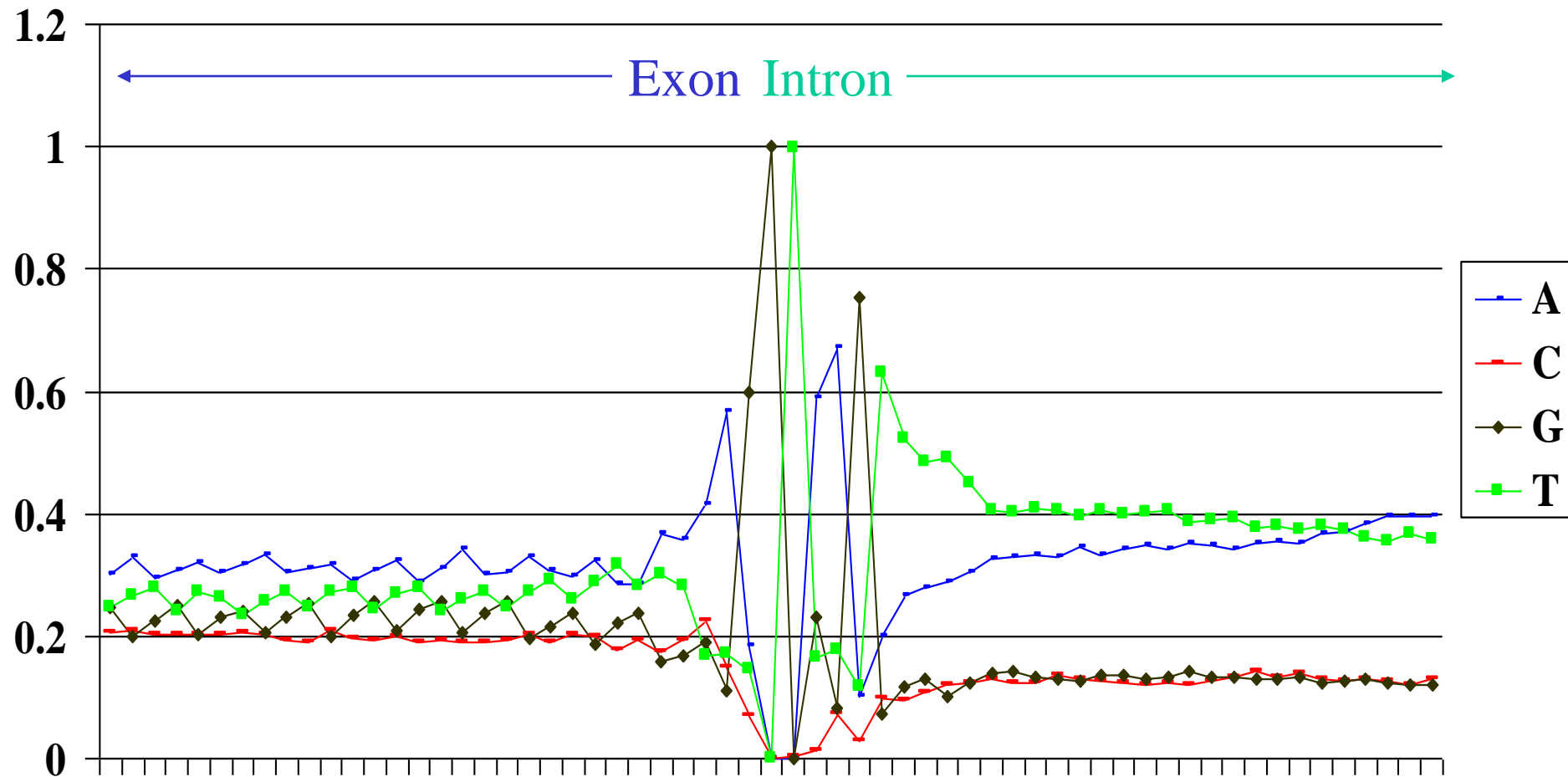
Nucleotide Counts for 8192 *C. elegans* 5' Splice Sites



A	3404	4644	1518	0	0	4836	5486	837	1632	2189	2278	2355
C	1850	1224	583	0	14	118	588	237	801	771	889	986
G	1562	912	4891	8192	0	1890	672	6164	589	962	1056	827
T	1376	1412	1200	0	8178	1348	1446	954	5170	4270	3969	4024

CONSENSUS	x	a	g	G	T	a	a	g	t	t	w	t
A	0.416	0.567	0.185	0.000	0.000	0.590	0.670	0.102	0.199	0.267	0.278	0.287
C	0.226	0.149	0.071	0.000	0.002	0.014	0.072	0.029	0.098	0.094	0.109	0.120
G	0.191	0.111	0.597	1.000	0.000	0.231	0.082	0.752	0.072	0.117	0.129	0.101
T	0.168	0.172	0.146	0.000	0.998	0.165	0.177	0.116	0.631	0.521	0.484	0.491

5' Splice Sites – *C. elegans*



Conserved Domain in RecR and Class I Topoisomerases

RecR RLAE EKITEVILATNPTVEGEATANYIAELC
 RecM RLQDDQVTEVILATNPNIERGEATAMYISRLL
 RecR RVDDVGITEVILATDPNTEGEATATYLVVMV
 TrsI IFKENKIDEVILATDPAREGENIAYKILNQL
 TOP1 KQLAEKADHIYLATDL DREG EAI AWRLREVI
 ORF1 AELLKQANTIIVATDS DREG ENIAWSIIHKA
 TOP1 KDALKDADELILATDE DREG KVISWHLLQLL
 TOP1 TIFDKRVKTIILATDAAAEGEYIGRNILYRL
 TOP3 KREARNADYLMIWTD CDREG EYIGWEIWQEA
 TOP3 KRFLHEASEIVHAGDP DREG QLLVDEVLDYL
 RGYR RNLAVEADEVLIGTDPDTEGEKIAWDLYLAL

CONSENSUS **xxxxxxxxxxU&uatDxxxEGexxxxxUxxxu**

Consensus key:

Uppercase: all residues chemically similar

lowercase: most are

U,u: bulky aliphatic (I,L,V)

&: bulky hydrophobic (I,L,V,M,F,Y,W)

From RL Tatusov, SF Altschul, and EV Koonin, PNAS 91: 12091-12095

Probability Models for Sites (assuming independence!)

- For each position i , $1 \leq i \leq n$, let P_i be a prob dist'n on the alphabet of residues
 - e.g. constructed using counts at that position in a sample of sites.
 - $P_i(r)$ for each residue r is the probability that r occurs at position i in a sequence.
- Prob dist'n P on the space S of sequences of length n is defined by

$$P(s) = \prod_{1 \leq i \leq n} P_i(s_i)$$

where $s = s_1 s_2 \dots s_n$

Zero Probabilities

- If $P_i(r) = 0$ for some i and r , then $P(s) = 0$ for some sequences.
 - may or may not be desirable
- If due to failure to observe residue because of small sample size,
 - should perform “small-sample correction” to change $P_i(r)$ to a small non-zero value.
 - usually done by adding ‘pseudocounts’ to each value in the counts matrix;
 - e.g. add 1 to each cell (has justification in Bayesian statistics)
 - Particularly an issue with proteins, due to larger alphabet size.
- If reflects real biological constraints
 - then leave as 0.
 - e.g. requirement for G at position +1 (first intronic base) in 5' ss

Independence assumption failures for Site Models

- 5' sites (Burge-Karlin observation)
- Offsetting changes for interacting residues
 - RNA stems,
 - protein motifs

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G	0.191	0.111	0.597	1.000	0.000	0.231	0.082	0.752	0.072	0.117	0.129	0.101
T	0.168	0.172	0.146	0.000	0.998	0.165	0.177	0.116	0.631	0.521	0.484	0.491

Failure of independence for 5' splice sites: G vs. H ('not G') at position -1

H in position -1 :

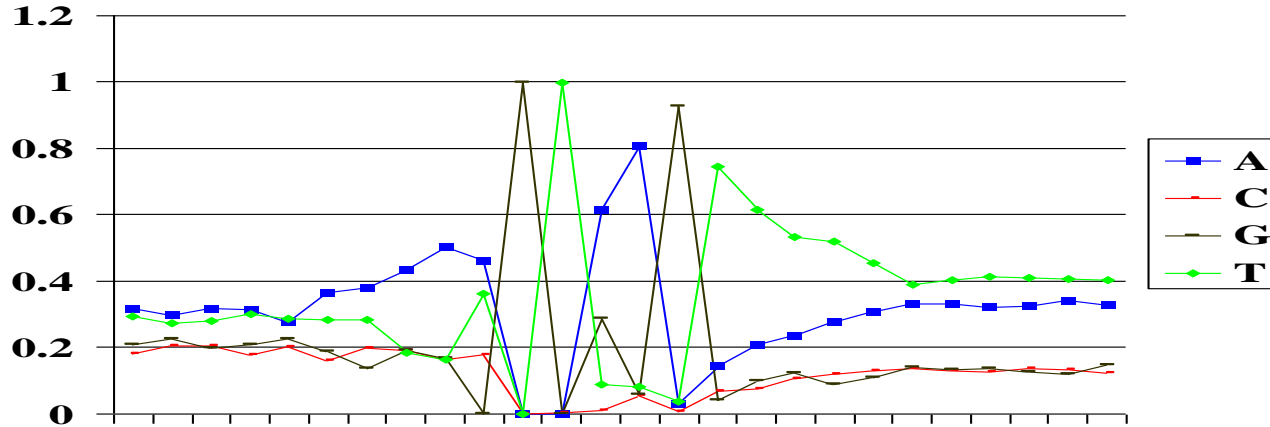
A	1434	1664	1518	0	0	2032	2662	98	479	694	783	912
C	633	546	583	0	5	36	177	22	225	250	350	393
G	628	553	0	3301	0	943	187	3063	134	329	405	279
T	606	538	1200	0	3296	290	275	118	2463	2028	1763	1717
A	0.434	0.504	0.460	0.000	0.000	0.616	0.806	0.030	0.145	0.210	0.237	0.276
C	0.192	0.165	0.177	0.000	0.002	0.011	0.054	0.007	0.068	0.076	0.106	0.119
G	0.190	0.168	0.000	1.000	0.000	0.286	0.057	0.928	0.041	0.100	0.123	0.085
T	0.184	0.163	0.364	0.000	0.998	0.088	0.083	0.036	0.746	0.614	0.534	0.520

G in position -1 :

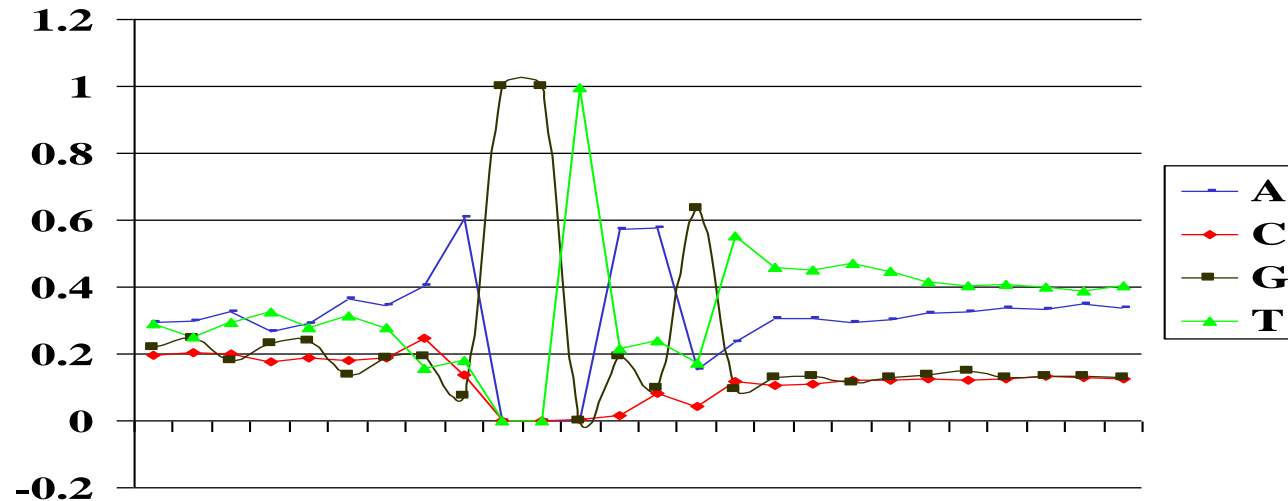
A	1970	2980	0	0	0	2804	2824	739	1153	1495	1495	1443
C	1217	678	0	0	9	82	411	215	576	521	539	593
G	934	359	4891	4891	0	947	485	3101	455	633	651	548
T	770	874	0	0	4882	1058	1171	836	2707	2242	2206	2307
A	0.403	0.609	0.000	0.000	0.000	0.573	0.577	0.151	0.236	0.306	0.306	0.295
C	0.249	0.139	0.000	0.000	0.002	0.017	0.084	0.044	0.118	0.107	0.110	0.121
G	0.191	0.073	1.000	1.000	0.000	0.194	0.099	0.634	0.093	0.129	0.133	0.112
T	0.157	0.179	0.000	0.000	0.998	0.216	0.239	0.171	0.553	0.458	0.451	0.472

5' Splice Sites – *C. elegans*

H at -1:



G at -1:



Why the correlation?

- Splicing involves pairing of a small RNA (U1 RNA) with the transcript at the 5' splice site (positions -2 to +7).
- The RNA is complementary to the 5' ss consensus sequence.
- A mismatch at position -1 tends to destabilize the pairing, & makes it more important for other positions to be correctly paired.

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T	0.168	0.172	0.146	0.000	0.998	0.165	0.177	0.116	0.631	0.521	0.484	0.491

complementary to portion of U1 RNA

Nucleotide Counts for 8192 *C. elegans* 3' Splice Sites

3' ss

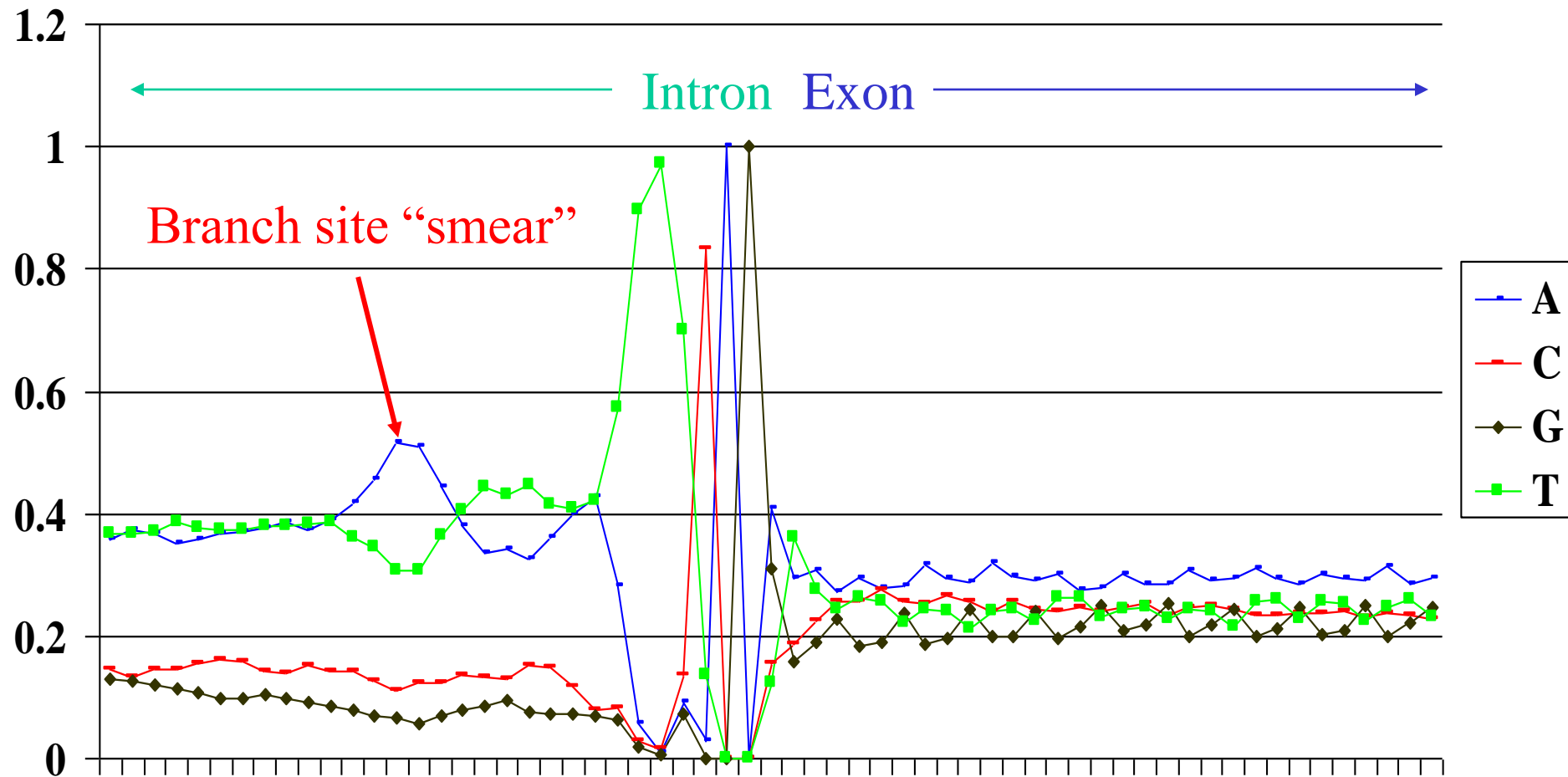


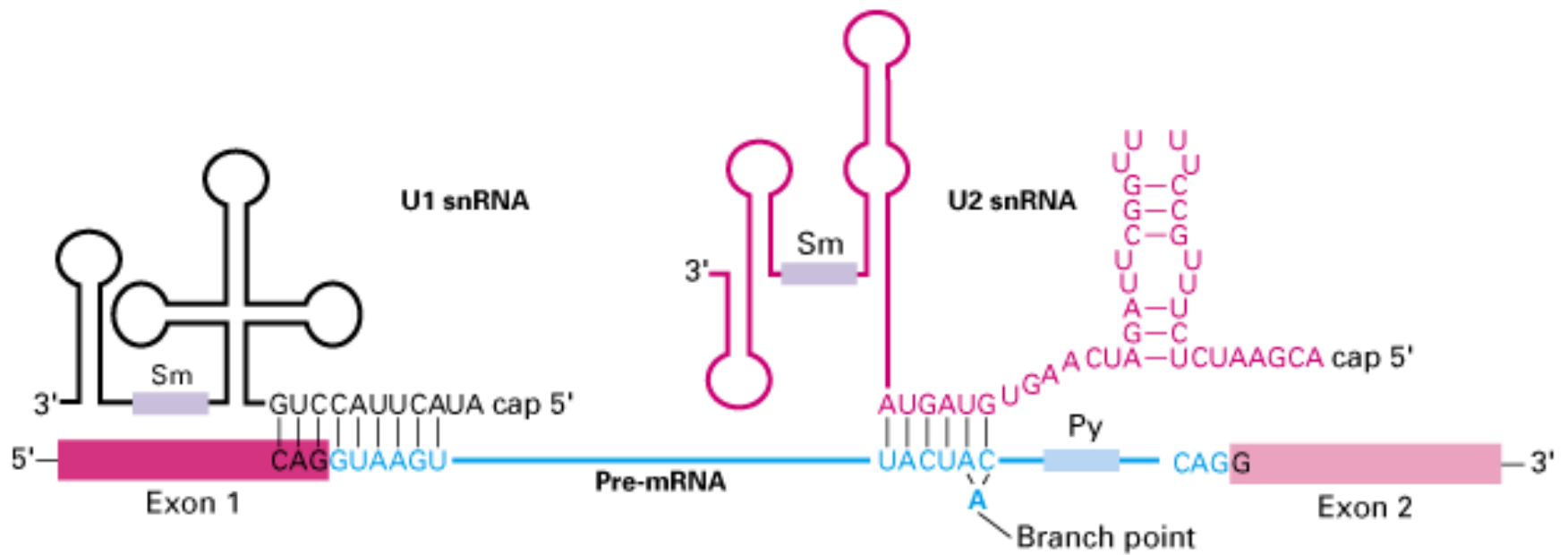
A	3276	3516	2313	476	67	757	240	8192	0	3359	2401	2514
C	970	648	664	236	129	1109	6830	0	0	1277	1533	1847
G	593	575	516	144	39	595	12	0	8192	2539	1301	1567
T	3353	3453	4699	7336	7957	5731	1110	0	0	1017	2957	2264

CONSENSUS W W W T T t C A G r w w

A	0.400	0.429	0.282	0.058	0.008	0.092	0.029	1.000	0.000	0.410	0.293	0.307
C	0.118	0.079	0.081	0.029	0.016	0.135	0.834	0.000	0.000	0.156	0.187	0.225
G	0.072	0.070	0.063	0.018	0.005	0.073	0.001	0.000	1.000	0.310	0.159	0.191
T	0.409	0.422	0.574	0.896	0.971	0.700	0.135	0.000	0.000	0.124	0.361	0.276

3' Splice Sites – *C. elegans*





from http://departments.oxy.edu/biology/Stillman/bi221/111300/processing_of_hnrnas.htm

(Jonathon Stillman, Grace Fisher-Adams)

- a 3' splice site includes more than one 'site'
(as we originally defined it)!